Downstream mergers in vertically related markets with capacity constraints

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Abstract

Motivated by a recent merger proposal in the French outdoor advertising market, we develop a model in which firms are initially endowed with some advertising capacities and compete on two fronts. First, firms compete to acquire additional advertising capacities on an upstream market; a first stage modeled as a second-price auction with externalities. Second, those firms, privately informed on their own costs, use their capacities on the downstream market to supply advertisers whose demand is random; a second stage modeled by means of mechanism design techniques. We study the linkages between the equilibrium outcomes on both markets. When a firm is endowed with more initial capacity, through the acquisition of a competitor for instance, whether it becomes more or less eager to acquire extra capacity on the upstream market depends a priori on fine details of the downstream market. Under reasonable choices of functional forms, we demonstrate that a downstream merger does not create any bias in the upstream market towards the already dominant firm.

Keywords: Merger, vertically related markets, competition with capacity constraints.

JEL Code: L1, D4, D8.

1. Introduction

An extensive literature studies how a stronger market power on the upstream market impacts the downstream market, yet less attention is paid to the reverse question, namely in what manner a stronger market power on the downstream market affects the upstream market. Motivated by a recent merger proposal in the French outdoor advertising market, we are particularly interested in understanding how a downstream merger impacts competition on the upstream market to acquire capacities.

In 2015, JCDecaux filed a merger application for the acquisition of Metrobus. Both firms were operating in the French market for outdoor advertising, which is one of the

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most traditional ways to market products and services. Outdoor advertising includes billboard advertising, street furniture, and transit advertising, such as mobile billboards found on buses, for instance. Although other types of traditional advertising have been in decline since the rise of online advertising, outdoor advertising was continuing to grow in a highly competitive environment thanks to new forms of display, such as LED screens.

Outdoor advertising is a vertically related industry. On the upstream market, firms acquire advertising spaces that are typically sold through tenders. These 'advertising capacities' are then packaged and sold on a downstream market to meet the advertisers' specific needs. For instance, an advertiser aiming to reach a business audience may be willing to advertise in major airports and train stations, while a restaurant may be more interested in advertising locally.

Metrobus was specialized on advertising in underground railways and train stations. JCDecaux and Clear Channel Outdoor - the largest player in the French market and its main competitor, respectively - were rather absent from this segment of outdoor advertising. Thus, the JCDecaux-Metrobus merger proposal was expected to significantly increase JCDecaux's advertising capacity and market power on the downstream market. However, after the completion of a Phase II investigation launched by the French Competition Authority and the remedies proposed by JCDecaux, the merger proposal was abandoned. Although the debate showcased the well-known impact of a merger on the downstream market, another perhaps less straightforward concern was a possible extension of market power towards the upstream market, following the downstream merger. To what extent would the merger have also changed JCDecaux's incentives to acquire additional capacities on the upstream market?

To answer these questions, we proceed in two steps. First, we suppose that, by acquiring Metrobus, JCDecaux would secure more initial capacity yet the structure of the downstream market would remain unchanged. Specifically, we assume that Metrobus was not a direct competitor to either JCDecaux or Clear Channel Outdoor on the downstream market. Whether Metrobus was on a substitutable or a complementary market was, however, a point of contention during the investigation. On the one hand, although Metrobus was present in two distinctive submarkets (advertising in metro and train stations) that targeted specific audiences (e.g., professionals), these audiences were also targeted by JCDecaux in other segments (e.g., advertising in airports). On the other hand, advertising in these two submarkets may have been used in combination with other advertising channels to promote global campaigns. It was therefore difficult to assess whether these firms are competitors or complementors.

Our analysis shows that, as the dominant firm enjoys more initial capacity, it does not necessarily behave more aggressively to acquire additional capacity on the upstream market. Whether this is the case depends largely on fine details of the downstream market. Yet, using simple functional forms (which assume that firms have costs uniformly and independently distributed, a linear demand and exponential demand shocks on the downstream market), we show that the downstream merger does not increase the position of the dominant firm on the upstream market. This finding suggests that antitrust authorities may be comfortable in dealing with the consequences of a merger by adopting a restricted stance on what happens on the downstream market, and ignoring possible feedback effects on the upstream market.

In a second step, we extend the analysis and examine a merger concerning two direct competitors on the downstream market and then discuss how our results are modified in this specific case. It turns out that our findings are to a large extent robust to the increased complexity of the setting.

Our model considers a vertically related market whereby two firms, initially endowed with some capacities, compete on two fronts. On the upstream market, those firms first compete to acquire extra capacities, and on the downstream market, they also compete to supply advertisers. Firms are asymmetric in terms of their initial capacities, such that the dominant firm has a larger capacity than the weak firm.

On the downstream market, a representative advertiser views the firms' advertising capacities as perfect substitutes. The advertiser demand is random and the firms have private information about their marginal cost. Efficiency requires that the firm with the lowest marginal cost supplies all the advertiser's needs. The most efficient firm may be capacity-constrained, however, and in such case the least efficient firm supplies the advertiser's residual demand up to its own capacity. Rather than specifying a particular extensive form to describe the interaction between firms selling their advertising capacities on the downstream market, we take an alternative route and adopt a mechanism design approach to characterize Bayesian equilibrium outcomes on the downstream market. Following the mechanism design tradition (Myerson, 1982), this approach allows us to fully characterize all possible allocations (i.e., market shares and profits) that might arise on the downstream market at any Bayesian equilibrium, without specifying the underlying game forms which those firms might actually be playing (possibly allowing multiple rounds of communications and side-payments). As such, this approach is powerful both from a theoretical and from a practical viewpoint. On the theory side, it allows us to handle in a very tractable way the possibility that firms are privately informed by means of simple Bayesian incentive compatibility constraints. On the practical side, this approach also offers powerful guidance for competition policy authorities, since the latter are likely to ignore fine details on the extensive form that firms are actually playing.

Of course, and it is a well-known aspect of mechanism design when compared with more direct game-theoretic approaches, the choice of an allocation on the downstream market follows from the maximization of a specific criterion. For most of our analysis, we assume that firms compete fiercely on the downstream market so that the induced equilibrium allocation maximizes the advertiser expected surplus (subject to incentive compatibility and participation constraints, of course). Yet, our results are robust to other specifications that could account for more collusive downstream behavior. We view this robustness as particularly appealing for antitrust authorities when in quest for robust responses.

The mechanism design approach allows us to derive the firms' downstream profits in terms of their initial advertising capacities. These reduced forms satisfy quite intuitive properties. A firm's profit is increasing in its own initial capacity and decreasing in that of its rival. It would then be tempting to conclude that a merger that increases a firm's capacity makes that firm more eager to buy additional capacities on the upstream market, leading to a snowball effect. That reasoning is incomplete, though, because it does not properly take into account the functioning of the upstream market and how a firm's incentives to acquire extra capacity are determined.

The upstream market is modeled as a second-price auction whereby firms bid to acquire some additional capacity. How much a firm is willing to pay for that extra capacity depends on how it impacts competition on the downstream market, both if the firm or instead its rival gets the extra capacity. The upstream auction is therefore an auction with externalities. The willingness to pay of a given firm is actually the difference between the firm's profit if it wins the auction, and its profit if it loses the auction and instead its competitor acquires the additional capacity.

We show that the willingness to pay of the larger firm is decreasing in its own capacity. This firm has therefore less incentives to acquire extra capacity as its own capacity grows. The intuition is that such an increase impacts the firm's profit only when this dominant firm is also the most efficient firm downstream, but it is capacity-constrained in case the advertiser expresses a high demand. Having more capacity reduces the likelihood of such events. At the same time, the willingness of a weak firm to pay for extra capacity also decreases with the dominant firm's capacity. Indeed, the profit of the weak firm only depends on this capacity, and negatively so, when the dominant firm is capacity-constrained and must allow a smaller and less efficient firm supply the advertiser's residual demand.

Because the willingness of both dominant and weak firms to pay for additional capacity decreases with the capacity of the dominant firm, the impact of the merger on the upstream market is a priori ambiguous. Put differently, a merger that increases the capacity of the dominant firm does not necessarily imply that that firm becomes more aggressive and wins more often in the upstream auction for extra capacity. Any presumption of an extension of market power from the downstream to the upstream market following a merger, often referred to by practitioners as a 'snowball effect,' should then be viewed with a word of caution. We use a particular specification of the model that assumes marginal costs are uniformly distributed, demand shocks are exponentially distributed and a linear demand. We show that the characteristics of the downstream market, such as the slope of the advertiser's demand and the distribution of the shock impacting the latter's demand, are key drivers for understanding the impact of the merger on the functioning of the upstream market. Nevertheless, this specification also shows that there is not necessarily an extension of market power towards the upstream market following a downstream merger.

Our welfare analysis reveals that the merger benefits the advertiser. On the one hand, the merger increases the average capacity available in the market, which ultimately benefits the advertiser and reduces the likelihood of firms being capacity-constrained. On the other hand, the merger also increases the asymmetry between firms, which negatively affects the advertiser. The former effect dominates in our particular specification of the model, which explains why the merger ends up benefiting the advertiser.

Finally, we also analyze a situation where, before the merger, three firms compete on the downstream market, and the merger involves two of those. Put differently, the merger now also impacts the market structure on the downstream market. Our findings are robust to that increased complexity of the setting, although the impact of the merger on the upstream market now sometimes depends on the degree of asymmetry between the initial capacity levels of the firms.

Stahl (1988) provides an earlier analysis of a setting where firms compete both on an upstream market and on a downstream market. Firms bid to acquire a homogeneous good, which is then resold to consumers via price competition. In some circumstances (one winner takes all, even when tying), the output price may be excessive. Yanelle (1997) studies a related model in which banks and borrowers compete for funds, leading to a coordination problem between lenders. Fingleton (1997) considers that direct trade between upstream suppliers and downstream buyers is also possible. Although we share with those papers the topic of 'middlemen' competing both to acquire inputs and to sell outputs, our analysis differs in several ways. For instance, we assume that there is a fixed quantity of input available.

Ghemawat (1990) builds on Kreps and Scheinkman (1983)'s analysis of capacity choice followed by price competition to study how two firms, initially endowed with different capacity levels, compete to buy additional capacity. He shows that the initially larger competitor ends up absorbing all investment opportunities in order to keep product prices high. Eső, Nocke and White (2010) assume that firms are Cournot competitors on the downstream market and compete to buy capacity in an upstream market that allocates capacity efficiently. In Burguet and Sákovics (2017a), the input is provided competitively by a large number of small capacity-constrained suppliers, but the same supplier may be approached by multiple potential buyers simultaneously. Firms become more aggressive upstream in the attempt to foreclose their rivals, which leads to a higher input price and a larger downstream quantity. We differ from those papers on two main grounds. First, we are interested in a different set of issues, namely how a merger on the downstream market impacts competition on the upstream market. Second, instead of a priori specifying the nature of competition on the downstream market, we rely on a mechanism design approach to characterize outcomes on this market. As argued above, this approach is attractive from a practitioner's viewpoint since it provides a guide for decision-making that does not rely on the interactions' details, which are often impossible to grasp for Antitrust authorities.

Indeed, and this points at its theoretical benefits, the mechanism design approach implicitly presumes that all possible sorts of communication procedures can a priori be entertained by competitors, yet it does not specify any extensive form to model their interactions. Any equilibrium allocation is therefore necessarily bound to satisfy a set of Bayesian incentive compatibility constraints that fully characterize the set of incentive-feasible allocations. Our mechanism design approach thus proposes an alternative route to study price competition with capacity constraints, thereby avoiding some of the technical difficulties encountered by the extant literature. Indeed, and contrary to our model, this literature focuses on complete information environments for which pure strategy equilibria often fail to exist, and only mixed-strategy equilibria can be characterized in some structured environments; see, for instance, Kreps and Scheinkman (1983), Davidson and Deneckere (1986), Osborne and Pitchik (1986), Deneckere and Kovenock (1992, 1996), and Allen, Deneckere, Faith and Kovenock (2000), or Burguet and Sákovics (2017b) who show that a pure strategy equilibrium exists when firms can use personalized pricing. The mechanism design approach provides a somewhat more tractable characterization of downstream outcomes, which in turn allows to perform several important exercises of comparative statics (e.g., on the market structure).

A large body of research addresses how a stronger market power on the upstream market impacts the downstream market (see Rey and Tirole, 2007, for a recent appraisal of the debate between the viewpoints of the so-called 'traditional foreclosure theory' and 'Chicago school'). Although we do not consider vertical contracting and foreclosure, as mentioned above, we are interested in understanding how a stronger market power on the downstream market impacts the upstream market.

One building block of our analysis models the upstream market as an auction with externalities, a topic investigated previously by Jéhiel, Moldovanu and Stacchetti (1999) and Jéhiel and Moldovanu (2000) among others. Unfortunately, this abstract literature is of little relevance to provide guidance for Antitrust analysis in practice since, contrary to our analysis, it usually does not endogenize those externalities by means of profit functions inherited from downstream behavior. Assuming Cournot competition on the downstream market, McAfee (1998) finds that small, capacity-constrained firms might often outbid larger, unconstrained firms. Mayo and Sappington (2016) study the foreclosure incentives of rivals bidding for an input and competing downstream à la Hotelling. They are interested in conditions that ensure the welfare-maximizing allocation of inputs. Our modeling of the downstream market differs from this approach and we focus instead on a different issue, namely how the asymmetry between firms, in terms of their initial capacity levels, affects the incentives to bid for extra capacity.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes the outcome on the downstream market when those firms are endowed with some given capacities. Section 4 studies how the upstream auction for additional capacity is biased when the dominant firm's capacity increases exogenously. Section 5 considers the same issue, but when the dominant firm's capacity increases following the acquisition of another downstream competitor. All proofs are relegated to an Appendix.

2. Model

Two firms, denoted by F_0 and F_1, compete on two fronts. On the upstream market, those firms compete to acquire extra capacities. On the downstream market, those firms use their capacities to supply advertisers.

The firms have some initial capacities, respectively denoted by K_0 and K_1. Those capacities might have been acquired through past tenders or negotiations, for instance. One of the firm, say F_0, is endowed with a larger initial capacity than its rival, namely $K_0 \geq K_1$. Firm F_0 is called the 'dominant firm' and firm F_1 the 'weak firm.' Competition on the upstream market is modeled as a second-price auction in which k new units of capacity have to be allocated to either one of these firms.

On the downstream market, firms supply a representative advertiser who views firms' products as perfect substitutes. The demand expressed by the advertiser is given by $D(p)+\varepsilon$, where ε is a random shock and p is the price paid by the advertiser for advertising slots. The demand function D is decreasing, namely $D' < 0$ at all positive prices. We also define the corresponding surplus as $S(q-\varepsilon)$, where $S' = D^{-1}$. Last, the demand shock ε is drawn from a common knowledge and atomless distribution F, with an everywhere positive density $f = F'$ on a support $[0, \overline{\varepsilon}]$.

Firm F_i's marginal cost to supply the advertiser is denoted by θ_i and is private information. Marginal costs are drawn from the same common knowledge distribution G, with (strictly positive) density $g = G'$, and support $\Theta = [\underline{\theta}, \overline{\theta}]$. As usual in the literature on information economics, the monotone hazard rate property is assumed to hold, that is, $\frac{d}{d\theta_i}(G(\theta_i)/g(\theta_i)) \geq 0$.[1]

The firms' initial capacities suffice to allow each firm to supply the market at the competitive price level in the absence of any demand shock, i.e., $\min(K_0; K_1) \geq D(\underline{\theta})$.[2]

[1]Myerson (1981), Laffont and Martimort (2002).

[2]This assumption is innocuous and allows us to streamline the exposition.

Notice that when the demand shock is sufficiently large, the most efficient firm may not be able to supply all the advertiser's demand due to its capacity constraint.

Running Example. We illustrate some of our results with the following simple specification of our model. Demand is linear, $D(p) = a - bp$. The demand shock ε is distributed according to the exponential distribution with parameter λ, $F(\varepsilon) = 1 - e^{-\lambda\varepsilon}$. Marginal costs are distributed according to the uniform distribution on $[0, 1]$, $G(\theta) = \theta$. Note that a larger value of b means that the price elasticity of demand increases, and a larger value of λ implies that small demand shocks are more likely. The assumption on capacities then amounts to $\min(K_0; K_1) \geq a$. We view these assumptions as simple and attractive enough to provide a reasonably good appraisal of the robustness of our findings. The results obtained here should continue to hold for alternative specifications that are "close enough" to those in the running example.

Timing and Structure of the Game. The game form is made of two building blocks that respectively unfold as follows.

1. Upstream Second-Price Auction. Firms F_0 and F_1 bid in a second-price auction to acquire k additional units of capacity. If F_i wins the upstream auction, its capacity becomes $K_i + k$ whereas that of its competitor remains equal to K_{-i}. Valuations for the extra units are endogenous and depend on the profits made in the downstream market. Capacities are common knowledge.

2. Downstream Market Competition. First, F_i $(i = 0, 1)$ privately learns its marginal cost θ_i to supply the downstream market. Second, firms compete to supply the downstream advertiser's demand. This stage is modeled as a generalized bargaining game between this advertiser and the firms that takes place under asymmetric information. A firm stands ready to supply q_i units of the advertiser's needs (when they realize) up to its own capacity at a predetermined payment t_i. Third, the demand shock ε realizes and becomes common knowledge. Each firm is paid for its deliveries.

The extant IO literature has developed a variety of models to understand how capacity constraints impact competition. Rather than specifying a particular extensive form, we remain agnostic on the nature of downstream competition. Following Myerson and Sattherwaite (1983), we rely on a mechanism design approach to characterize all possible outcomes that might arise at any Bayesian-Nash equilibrium on the downstream market by means of simple Bayesian incentive compatibility and participation constraints. According to the Revelation Principle,[3] there is no loss of generality in representing such allocations by means of truthful and direct revelation mechanisms of the form $(t_i(\hat{\theta}_i, \hat{\theta}_{-i}), q_i(\hat{\theta}_i, \hat{\theta}_{-i}, \varepsilon))_{(\hat{\theta}_i, \hat{\theta}_{-i}) \in \Theta^2}$, $i = 0, 1$, where t_i is the payment from the advertiser to firm F_i in exchange of a quantity q_i when firm F_i reports a cost $\hat{\theta}_i$, firm F_{-i} a cost $\hat{\theta}_{-i}$ and the demand shock that realizes is ε.[4] A feasible contract must also give positive profits to the firms and ensure that each firm has incentives to report truthfully its

[3]Myerson (1982).

[4]The quantities bought by the advertiser at each firm may a priori depend on the random shock on demand ε. Because all parties are risk neutral, there is no need to condition transfers on the realization of the demand shock. Any transfer schedule $\tilde{t}_i(\hat{\theta}_i, \hat{\theta}_{-i}, \varepsilon)$ that would make payments explicitly dependent on the realization of ε could be replaced by its expectation, namely $t_i(\hat{\theta}_i, \hat{\theta}_{-i}) = \mathbb{E}_\varepsilon(\tilde{t}_i(\hat{\theta}_i, \hat{\theta}_{-i}, \varepsilon))$ without changing the incentive properties of the mechanism and without affecting payoffs. Of course, quantities should depend on ε to respect capacity constraints.

marginal cost. Thanks to their owning private information, firms get some information rents at this stage and leave the residual expected surplus to the advertiser.

Our analysis proceeds from now on in two steps. First, Section 3 offers a complete characterization of all possible allocations (that is, quantities and profits) that may arise at any Bayesian-Nash equilibrium on the downstream market stage. Second, Section 4 selects one particular allocation within that set, namely the allocation that maximizes the advertiser surplus.

Our objective is to understand how an increase in the capacity K_0 held by the dominant firm F_0 impacts competition in the auction for additional capacities on the upstream market. Building on our motivating example, that increase in capacity K_0 may result from the merger between F_0 and another firm that does not directly compete with F_0 and F_1. Such a comparative statics is thus a shortcut to analyze the impact of the merger proposal of our motivating example on the functioning of the upstream market. Later on, Section 5 relaxes this assumption and considers a merger between two firms that are direct competitors on the downstream market.

3. Downstream Market Outcomes: A Mechanism Design Characterization

3.1. *Incentive-Feasible Allocations*

Let denote by $U(\theta_i)$ the expected profit of firm F_i with marginal cost θ_i on the downstream market. For ease of notations, we will for a while omit the dependency of this profit on the existing capacities. Bayesian incentive compatibility ensures that

$$U_i(\theta_i) = \max_{\hat{\theta}_i \in \Theta} \mathbb{E}_{\theta_{-i}}(t_i(\hat{\theta}_i, \theta_{-i})) - \theta_i \mathbb{E}_{(\theta_{-i},\varepsilon)}(q_i(\hat{\theta}_i, \theta_{-i}, \varepsilon)),$$

where the maximand is achieved when F_i follows a truthful strategy and always reveals its cost. Using the Envelope Theorem under those circumstances leads to the following condition

$$\dot{U}_i(\theta_i) = -\mathbb{E}_{(\theta_{-i},\varepsilon)}(q_i(\theta_i, \theta_{-i}, \varepsilon)). \tag{3.1}$$

Condition (3.1) has an intuitive interpretation. Firm F_i with marginal cost θ_i could mimic the strategy of a firm with a slightly higher marginal cost $\theta_i + d\theta_i$, thereby saving on cost approximately the amount $\mathbb{E}_{(\theta_{-i},\varepsilon)}(q_i(\theta_i, \theta_{-i}, \varepsilon))d\theta_i$. That firm must therefore be given up an incremental profit $U_i(\theta_i - d\theta_i) - U_i(\theta_i) \approx \dot{U}_i(\theta_i)d\theta_i$ worth such a cost saving in order to reveal truthfully its cost.[5]

Firms are willing to accept the mechanism provided that their gains are positive

$$U_i(\theta_i) \geq 0 \quad \forall \theta_i. \tag{3.2}$$

The Bayesian incentive compatibility constraint (3.1) imposes that the profile of gain $U_i(\cdot)$ is decreasing with F_i's marginal cost θ_i. The participation constraint defined in

[5]The familiar second-order condition for incentive compatibility, which boils down to the convexity of $U_i(\theta_i)$ (or alternatively the fact that $\mathbb{E}_{(\theta_{-i},\varepsilon)}(q_i(\theta_i, \theta_{-i}, \varepsilon))$ is non-increasing) is assumed to be satisfied in the sequel. One can check that it is indeed the case in our running example. Well-known ironing techniques could be used otherwise. We leave those technical developments aside for simplicity.

(3.2) is therefore always satisfied provided that it holds for the least efficient firm, i.e.,

$$U_i(\overline{\theta}) \geq 0. \tag{3.3}$$

Lastly, firm F_i's profit and expected profit are respectively given by

$$U_i(\theta_i) = U_i(\overline{\theta}) + \int_{\theta_i}^{\overline{\theta}} \mathbb{E}_{(\theta_{-i},\varepsilon)}(q_i(x,\theta_{-i},\varepsilon))dx \tag{3.4}$$

and

$$\mathbb{E}_{\theta_i}(U_i(\theta_i)) = U_i(\overline{\theta}) + \mathbb{E}_{(\theta_i,\theta_{-i},\varepsilon)}\left(\frac{G(\theta_i)}{g(\theta_i)}q_i(\theta_i,\theta_{-i},\varepsilon)\right). \tag{3.5}$$

3.2. Downstream Market Equilibrium

We consider an environment where downstream competition is fierce. Accordingly, we look for the allocation that maximizes the advertiser expected surplus. Such allocation therefore satisfies

$$\max_{(q_0(\cdot),q_1(\cdot),U_0(\cdot),U_1(\cdot))} \mathbb{E}_{(\theta_0,\theta_1,\varepsilon)}\left(S\Big(\sum_{i=0,1} q_i(\theta_i,\theta_{-i},\varepsilon) - \varepsilon\Big) - \sum_{i=0,1}\theta_i q_i(\theta_i,\theta_{-i},\varepsilon) - \sum_{i=0,1} U_i(\theta_i)\right),$$

subject to the incentive constraint (3.1), the participation constraint (3.3), and the constraint that each firm cannot produce more than its capacity

$$q_i(\theta_i,\theta_{-i},\varepsilon) \leq \hat{K}_i, \quad i = 0, 1,$$

where $\hat{K}_i$ denotes firm F_i's capacity after the upstream competition stage. Standard computations allow then to characterize optimal production profiles given firms' capacity constraints.

Proposition 1. *Given the capacities $(\hat{K}_i, \hat{K}_{-i})$ held by the firms after the upstream market stage, equilibrium quantities on the downstream market are given by*[6]

$$q_i(\theta_i,\theta_{-i},\varepsilon) = \begin{cases} \min(D(\tilde{\theta}_i)+\varepsilon;\hat{K}_i) & \text{if } \theta_i < \theta_{-i}\ , \\ \min(\max(D(\tilde{\theta}_i)+\varepsilon-\hat{K}_{-i},0);\hat{K}_i) & \text{if } \theta_i > \theta_{-i}, \end{cases}$$

where $\tilde{\theta}_i \equiv \theta_i + G(\theta_i)/g(\theta_i)$ is F_i's virtual marginal cost.

Observe that due to the monotone hazard property, virtual marginal costs are ranked as marginal costs, that is, $\theta_i < \theta_{-i} \Leftrightarrow \tilde{\theta}_i < \tilde{\theta}_{-i}$.

The expression for equilibrium quantities shows that the advertiser buys first from the firm with the lowest virtual marginal cost, which is the most efficient firm, a quantity corresponding to the demand expressed at that firm's virtual marginal cost. This is possible provided that the most efficient firm has enough capacity. When the shock on demand is sufficiently large, that firm cannot supply the whole demand expressed by

[6] In the zero-measure event where $\theta_i = \theta_{-i}$, quantities may be chosen so that $q_i = \min\{D(\tilde{\theta}_i)+\varepsilon;\hat{K}_i\}$ if $\theta_i = \theta_{-i}$ and $\hat{K}_i > \hat{K}_{-i}$; $q_i = \min\{D(\tilde{\theta}_i)+\varepsilon-\hat{K}_{-i};\hat{K}_i\}$ if $\theta_i = \theta_{-i}$ and $\hat{K}_i < \hat{K}_{-i}$; $q_i = \min\{\frac{1}{2}(D(\tilde{\theta}_i)+\varepsilon-\hat{K}_{-i});\frac{\hat{K}_i}{2}\}$ if $\theta_i = \theta_{-i}$ and $\hat{K}_i = \hat{K}_{-i}$.

the advertiser, who has to buy the residual demand from the least efficient firm. That residual demand is evaluated at the least efficient firm's virtual marginal cost. Again, this is possible only if the least efficient firm's capacity constraint is sufficiently large.

Implementation. The optimal allocation in Proposition 1 can be implemented on the downstream market with a simple game form. The advertiser runs a multi-unit auction with pecking order for its suppliers, in which the most efficient firm is called upon first up to its capacity before the least efficient one is asked to supply any residual demand. To see how this game form is played, let us first define an output profile $\hat{q}_i(\theta_i)$ as

$$S'(\hat{q}_i(\theta_i)) = \theta_i + \frac{G(\theta_i)}{g(\theta_i)}$$

and denote by $\vartheta(\hat{q}_i)$ the corresponding inverse function. Consider now the following family of nonlinear payment schedules, indexed by q^*_{-i} and ε, that the advertiser offers F_i before the realization of the demand shock ε and for each possible quantity q^*_{-i} that F_{-i} might put on the market

$$\tilde{S}(q_i + q^*_{-i} - \varepsilon) = S(q_i + q^*_{-i} - \varepsilon) - \int_0^{q_i + q^*_{-i} - \varepsilon} \frac{G(\vartheta(\tilde{q}_i))}{g(\vartheta(\tilde{q}_i))} d\tilde{q}_i + C_i,$$

where C_i is a constant to be soon defined. For each possible realization of q^*_{-i} and ε, and given its own capacity $\hat{K}_i$, F_i then stands ready to supply the quantity

$$\tilde{q}_i(\theta_i, q^*_{-i} - \varepsilon) = \min\{\max\{\tilde{D}(\theta_i) + \varepsilon - q^*_{-i}, 0\}, \hat{K}_i\}$$

where $\tilde{D} = \tilde{S}'^{-1}$. That quantity maximizes over $[0, \hat{K}_i]$ the profit of F_i (namely $\tilde{S}(q_i + q^*_{-i} - \varepsilon) - \theta_i q_i$) for each possible realization of the demand shock ε and for each possible quantity q^*_{-i} that F_{-i} might put on the market. The expression $\tilde{q}_i(\theta_i, q^*_{-i} - \varepsilon)$ can thus be viewed as F_i's best-response. At the last stage of the game, once the demand shock ε has realized, the rule of the game specifies that the advertiser picks the pair of quantities (q^*_0, q^*_1) that are at the intersection of the firms' best responses. Observe that, by construction, we have $\tilde{D}(\theta_i) = D(\tilde{\theta}_i)$. It is then straightforward to check that, at the equilibrium of this game, F_i produces the optimal quantity $q_i(\theta_i, \theta_{-i}, \varepsilon)$ characterized in Proposition 1. The corresponding payments are then made to the firms. Constant C_i is chosen so as to ensure that the least-efficient type of F_i breaks even in expectations.

Expected Profits. Using Condition (3.4) and the equilibrium quantities stated in Proposition 1, we can determine F_i's profit or information rent, which depends on the capacities $(\hat{K}_i, \hat{K}_{-i})$

$$\begin{aligned}
(3.6) \qquad U_i(\theta_i) &= \int_{\theta_i}^{\overline{\theta}} \mathbb{E}_{(\theta_{-i},\varepsilon)}(q_i(x, \theta_{-i}, \varepsilon))dx, \\
&= \int_{\theta_i}^{\overline{\theta}} \Bigg((1 - G(x)) \int_0^{\overline{\varepsilon}} \min(D(\tilde{\theta}(x)) + \varepsilon; \hat{K}_i) f(\varepsilon) d\varepsilon \\
&\qquad + G(x) \int_{\hat{K}_{-i} - D(\tilde{\theta}(x))}^{\overline{\varepsilon}} \min(D(\tilde{\theta}(x)) + \varepsilon - \hat{K}_{-i}; \hat{K}_i) f(\varepsilon) d\varepsilon \Bigg) dx,
\end{aligned}$$

where $\tilde{\theta}(x)$ stands for the virtual marginal cost associated to marginal cost x.

Using (3.6), F_i's expected profit is then given by

$$\begin{aligned}(3.7)\qquad V_i(\hat{K}_i, \hat{K}_{-i}) &\equiv \mathbb{E}_{\theta_i}(U_i(\theta_i)) \\ &= \mathbb{E}_{\theta_i}\left(\frac{G(\theta_i)}{g(\theta_i)}\left((1-G(\theta_i))\int_0^{\bar{\varepsilon}} \min(D(\tilde{\theta}_i)+\varepsilon; \hat{K}_i) f(\varepsilon)d\varepsilon \right.\right. \\ &\qquad \left.\left. + G(\theta_i)\int_{K_{-i}-D(\tilde{\theta}_i)}^{\bar{\varepsilon}} \min(D(\tilde{\theta}_i)+\varepsilon-\hat{K}_{-i}; \hat{K}_i) f(\varepsilon)d\varepsilon\right)\right).\end{aligned}$$

Given the firms' capacities, we have fully characterized quantities and profits that emerge from the interaction between firms on the downstream market. Before moving on to the analysis of the upstream market equilibrium, the next proposition provides useful comparative statics on ex ante profit function $V_i(\hat{K}_i, \hat{K}_{-i})$.

PROPOSITION 2. *The ex ante profit $V_i(\hat{K}_i, \hat{K}_{-i})$ is*

- *strictly increasing and strictly concave in $\hat{K}_i$;*
- *strictly decreasing in $\hat{K}_{-i}$.*

The first property is intuitive. When F_i has more capacity, it can supply a larger demand even if it is not the most efficient firm and supplies only the advertiser's residual demand when F_{-i} is capacity-constrained. The second property showcases the existence of decreasing returns to scale. The incremental profits earned from having additional capacities are decreasing with the stock of capacity already owned. The third property is best understood by noticing that the rival's capacity $\hat{K}_{-i}$ affects F_i only when F_i is the least efficient supplier and supplies a residual demand that decreases with the most efficient firm F_{-i}'s capacity.

4. UPSTREAM COMPETITION

4.1. Main Analysis

Upstream competition for extra capacity is modeled as a second-price auction in which firms F_0 and F_1 bid to acquire k additional units of capacity.[7] Given that bidders in the auction are also competitors in the downstream market, a firm that loses the auction is put at a competitive disadvantage on the downstream market. The upstream auction is therefore an auction with externalities. How much a firm is willing to pay for extra capacity depends on the comparison between its downstream profit if it wins the auction, namely $V_i(K_i+k, K_{-i})$, and its profit if it loses the auction and its competitor gets those k extra units, namely $V_i(K_i, K_{-i}+k)$. F_i's willingness to pay for k additional units of capacities is thus given by

$$V_i(K_i+k, K_{-i}) - V_i(K_i, K_{-i}+k).$$

The second-price auction is efficient. Bidding competition then leads to allocate the k units to firm F_0 when

$$(4.1)\qquad V_0(K_0+k, K_1) - V_0(K_0, K_1+k) \geq V_1(K_1+k, K_0) - V_1(K_1, K_0+k),$$

[7]Since we are interested in how a change in initial capacities affects the outcome of the upstream auction, and more precisely whether the dominant firm F_0 wins more or less often that auction when its initial capacity increases, our analysis carries over to other auction formats as long as they are efficient.

and to firm F_1 otherwise.

In the remainder, we analyze how the dominant firm's initial capacity K_0 impacts the upstream auction. To do so, we shall compare how a change in K_0 affects both the dominant firm's willingness to pay $V_0(K_0+k, K_1) - V_0(K_0, K_1+k)$ and the weak firm's willingness to pay $V_1(K_1+k, K_0) - V_1(K_1, K_0+k)$. So doing will allow to understand how the allocation rule given by Equation (4.1) is modifed when K_0 varies.

From now on, we shall say that a merger that increases the dominant firm F_0's initial capacity K_0 biases the allocation of extra capacity in the upstream auction towards the dominant firm F_0 when

$$\frac{\partial}{\partial K_0}\left[(V_0(K_0+k, K_1) - V_0(K_0, K_1+k)) - (V_1(K_1+k, K_0) - V_1(K_1, K_0+k))\right] > 0,$$

that is, when F_0 wins more often the upstream auction after the merger than before.

We start by describing how the firms' willingnesses to pay for such extra capacity depend on the current level of capacity.

PROPOSITION 3. *Willingnesses to pay for additional capacity k are such that*

- *the dominant firm F_0's willingness to pay decreases with its own capacity K_0,*

$$\frac{\partial}{\partial K_0}\big(V_0(K_0+k, K_1) - V_0(K_0, K_1+k)\big) < 0;$$

- *the weak firm F_1's willingness to pay decreases with the capacity K_0 of its rival,*

$$\frac{\partial}{\partial K_0}\big(V_1(K_1+k, K_0) - V_1(K_1, K_0+k)\big) < 0.$$

As its capacity K_0 grows, the dominant firm F_0 has less incentives to acquire additional capacities on the upstream market. Yet, to assess the impact on upstream competition, we must also evaluate the bidding incentives of F_1, which is endowed with a smaller capacity. Importantly, the weak firm F_1's willingness to pay also decreases when the dominant firm's capacity K_0 increases. The impact of an increase in K_0 on the upstream market remains thus a priori ambiguous.

To describe the intuition underlying Proposition 3, consider the impact on profit of a marginal increase in the dominant firm's capacity K_0 (remind that $\tilde{\theta}_0$ is the virtual marginal cost corresponding to θ_0)

$$\frac{\partial V_0}{\partial K_0}(K_0, K_1) = \mathbb{E}_{\theta_0}\left(\frac{G(\theta_0)}{g(\theta_0)}\left(\underbrace{(1-G(\theta_0))(1-F(K_0-D(\tilde{\theta}_0)))}_{F_0 \text{ is more efficient downstream}} + \underbrace{G(\theta_0)(1-F(K_0+K_1-D(\tilde{\theta}_0)))}_{F_0 \text{ is less efficient downstream}}\right)\right) > 0. \tag{4.2}$$

The first term on the right-hand side of Equation (4.2) is the marginal gain of increasing K_0 when F_0 is the most efficient firm downstream, which explains the weight $1 - G(\theta_0)$ corresponding to the probability of such an event. Marginally increasing capacity K_0 increases the profit earned when F_0's capacity is binding by $1 - F(K_0 - D(\tilde{\theta}_0))$.

The second term is the dominant firm's marginal gain of increasing capacity when it is less efficient than its rival on the downstream market, which happens with probability $G(\theta_0)$. F_0 only gains when the demand shock is sufficiently strong so that the efficient firm F_1 cannot supply all the advertiser's demand, which happens with probability $1 - F(K_0 + K_1 - D(\tilde{\theta}_0))$.

Observe that the second term in Equation (4.2) only depends on the sum of the capacities held by both firms. This term is, therefore, not impacted by the allocation of extra capacity in the upstream auction. Put differently, whether the extra capacity is obtained by firm F_0 or firm F_1, the value of such additional capacity for F_0 is the same conditional on the fact that F_0 is less efficient and supplies only the advertiser's residual demand.

Consequently, the marginal impact of K_0 on F_0's willingness to pay depends only on how these capacities impact its profit when F_0 is the most efficient firm on the downstream market. This implies that F_0's willingness to pay for k additional units of capacity decreases with its own capacity K_0 because capacity constraints are less often binding in those events. Using Equation (4.2), we get

$$\begin{aligned}\frac{\partial}{\partial K_0}\big(V_0(K_0+k, K_1) - V_0(K_0, K_1+k)\big) = \\ \mathbb{E}_{\theta_0}\left(\frac{G(\theta_0)(1-G(\theta_0))}{g(\theta_0)}(F(K_0 - D(\tilde{\theta}_0)) - F(K_0 + k - D(\tilde{\theta}_0)))\right) < 0.\end{aligned} \tag{4.3}$$

Next, consider the second item in Proposition 3, which describes the impact of an increase in the dominant firm's capacity K_0 on the rival firm's profit. The effect is more subtle and relates to the existence of an externality between firms on the downstream market. F_0's capacity K_0 has no impact on F_1's profit when F_1 is the most efficient firm on the downstream market. Capacity K_0 impacts F_1 when F_1 supplies the advertiser's residual demand because the most efficient firm F_0 is capacity-constrained. Increasing K_0 decreases the probability of such an event and has thus a negative impact on F_1's willingness to pay. Formally, we obtain

$$\begin{aligned}\frac{\partial}{\partial K_0}\big(V_1(K_1+k, K_0) - V_1(K_1, K_0+k)\big) = \\ \mathbb{E}_{\theta_1}\left(\frac{G^2(\theta_1)}{g(\theta_1)}\big(F(K_0 - D(\tilde{\theta}_1)) - F(K_0 + k - D(\tilde{\theta}_1))\big)\right) < 0.\end{aligned} \tag{4.4}$$

Coming back to our motivating example, a merger that increases the dominant firm F_0's capacity does not systematically make that firm more eager to acquire extra capacity in the upstream market. The capacity obtained through the merger softens the dominant firm F_0's behavior on the upstream auction because it decreases its willingness to pay in that auction. The merger also reduces the willingness to pay of the weak rival F_1, though, making that firm softer in the auction.[8] Overall, the impact of the merger on the upstream auction is ambiguous. The next proposition gives a condition under which the upstream auction is biased towards either the dominant firm F_0 or the weak firm F_1.

[8] Fudenberg and Tirole (1984).

PROPOSITION 4. *When the dominant firm F_0's capacity K_0 increases, the upstream auction for k units of extra capacity is biased towards F_0 if and only if*

$$\mathbb{E}_\theta\left(\frac{G(\theta)(1-2G(\theta))}{g(\theta)}\big(F(K_0+k-D(\tilde{\theta}))-F(K_0-D(\tilde{\theta}))\big)\right)<0. \tag{4.5}$$

Consider now the case of a small extra capacity k. Condition (4.5) is thus replaced by a simpler marginal condition

$$\mathbb{E}_\theta\left(\frac{G(\theta)(1-2G(\theta))}{g(\theta)}f(K_0-D(\tilde{\theta}))\right)<0. \tag{4.6}$$

Assume that the density $f(\cdot)$, the capacity K_0 and the downstream demand $D(\cdot)$ are such that $f(K_0-D(\tilde{\theta}_0))\approx 0$ for θ_0 larger than the median value associated to distribution $G(\cdot)$, which requires that $f(\cdot)$ be decreasing for sufficiently strong demand shocks. Condition (4.6) cannot be satisfied and the auction is not biased towards the dominant firm F_0.

4.2. Discussions and Extensions in the Running Example

Auction Bias. To further illustrate the discussion above, consider our running example. Condition (4.5) thus amounts to

$$\frac{-1}{4b^3\lambda^3}\underbrace{e^{(a-2b-k-K_0)\lambda}(e^{k\lambda}-1)}_{\text{intensity of the bias}}\underbrace{\left(e^{2b\lambda}(-2+b\lambda)+2+3b\lambda+2(b\lambda)^2\right)}_{\text{nature of the bias}}>0 \tag{4.7}$$

Condition (4.7) shows that whether the upstream auction is biased in favor or against the dominant firm depends entirely on characteristics of the downstream market (namely the advertiser's demand and the distribution of shocks) but neither on the initial capacities held by firms nor the amount of extra capacity available on the upstream market. More precisely, the key parameter is $b\lambda$. Whether the upstream auction is biased towards the dominant firm depends on the slope of the advertiser's demand and on whether the distribution of demand shocks puts a large weight on high shocks. One can then show the following result.

LEMMA 1. *There exists a threshold $\underline{b\lambda}$ such that the upstream auction is biased towards the dominant firm if and only if $b\lambda\leq\underline{b\lambda}$.*

As λ decreases, the density of the demand shock (which is always decreasing) becomes flatter. When $\lambda\approx 0$, the term $F(K_0+k-D(\tilde{\theta}))-F(K_0-D(\tilde{\theta}))$ in Condition (4.5) becomes approximately equal to k. With a uniform distribution on $[0,1]$ for marginal costs, $\mathbb{E}_\theta(\frac{G(\theta)(1-2G(\theta))}{g(\theta)})<0$. This explains why Condition (4.5) is satisfied when λ is sufficiently small. A similar intuition obtains when the downstream demand is sufficiently inelastic. As b goes to 0, the term $F(K_0+k-D(\tilde{\theta}))-F(K_0-D(\tilde{\theta}))$ converges to $(e^{\lambda k}-1)e^{(a-k-K_0)\lambda}>0$. Thus, in the running example, the upstream auction is biased towards the dominant firm following the merger when the demand on downstream market is inelastic and the distribution of demand shocks puts enough weight on large shocks.

The dominant firm's capacity K_0 does not affect the nature of the bias in the upstream auction. Put differently, the acquisition of capacity through the merger has no impact on the sign of the bias in the upstream auction. The same result holds for the size k of the additional capacity to be allocated on the upstream market. These two variables have an impact on the intensity of the bias in the upstream auction, but not on its direction. Thus, if the upstream auction was biased before the merger towards one firm, it remains biased in the same direction after the merger.

Consequence of the Auction Bias on the Advertiser Expected Surplus. Our running example can also be used to analyze how the bias that affects the upstream auction impacts the adviser expected surplus.

Before doing so, we study how the asymmetry between firms with respect to their capacity levels impacts the advertiser expected surplus. Suppose capacities are (K_0, K_1) and let $\Delta K = (K_0 - K_1)/2$ and $K = (K_0 + K_1)/2$ to illustrate the role of the asymmetry in the firms' capacity levels and the average capacity available in the market. The dominant firm's and the weak firm's capacities may thus be rewritten as $K_0 = K + \Delta K$ and $K_1 = K - \Delta K$ respectively. The advertiser expected surplus is then denoted by $CS(K + \Delta K, K - \Delta K)$.[9]

LEMMA 2. *The advertiser expected surplus is increasing in the average capacity K and decreasing in the difference in capacities ΔK*

$$\frac{\partial}{\partial \Delta K} CS(K + \Delta K, K - \Delta K) < 0 < \frac{\partial}{\partial K} CS(K + \Delta K, K - \Delta K),$$

with $|\frac{\partial}{\partial \Delta K} CS(K + \Delta K, K - \Delta K)| \leq \frac{\partial}{\partial K} CS(K + \Delta K, K - \Delta K)$.

That the advertiser expected surplus increases with the average capacity is intuitive. When, on average, firms are endowed with more capacity, they will produce more to the benefit of the advertiser. The role of the asymmetry between firms, in terms of their capacities as measured by ΔK, is more interesting. Indeed, the advertiser dislikes being served at a higher price by the least efficient firm when the most efficient one is capacity-constrained. In a rough sense, moving towards a more symmetric allocation of capacities means reducing such risk exposure and increases the advertiser expected surplus.

We can now study how the upstream auction bias impacts the advertiser expected surplus.

LEMMA 3. *The advertiser expected surplus is*

- *larger when the weak firm F_1 wins the upstream auction,*

$$CS(K_0, K_1 + k) \geq CS(K_0 + k, K_1);$$

- *increasing with the dominant firm F_0's capacity,*

$$\frac{\partial}{\partial K_0} CS(K_0, K_1 + k) \geq 0 \text{ and } \frac{\partial}{\partial K_0} CS(K_0 + k, K_1) \geq 0.$$

As Lemma 2 has shown, the advertiser surplus is larger when firms become more symmetric in terms of their capacity levels. When the weak firm F_1 wins the upstream auction, capacities become K_0 and $K_1 + k$, whereas they are given by $K_0 + k$ and K_1 when the dominant firm F_0 instead wins. That F_1 wins benefits the advertiser because

[9] By analogy with our writing of profit functions, the first and second arguments in the advertiser surplus CS correspond to, respectively, F_0's and F_1's capacity levels.

firms thus become more alike. Thus, from the advertiser's viewpoint, it would be better that the weak firm F_1 wins the extra capacity, which arises only provided that $b\lambda \geq \underline{b\lambda}$ according to Lemma 1.

Remind that, in the running example, a merger that increases the dominant firm F_0's capacity K_0 does not change the nature of the auction bias. Thus, the second item in Lemma 3 shows that the merger always improves the advertiser expected surplus. A merger has a priori a positive and a negative effect on this surplus. It increases the average capacity available in the market, but it also increases asymmetry between firms. From Lemma 2, the positive effect of having more capacities in the market outweighs the negative effect of having more asymmetric firms.

Comparison with Cournot Competition on the Downstream Market. One may wonder how our characterization of downstream outcomes by means of mechanism design compares with the outcomes obtained under a more explicit modeling of the game form that rules downstream competition. There are no straightforward answers to such question, for there are many ways to model competition between firms that face capacity constraints. In the Appendix, we develop a simple model of Cournot competition with differentiated products and capacity constraints on the downstream market. The insights obtained from such model echo those obtained in our main analysis. Both the weak and the dominant firm's willingnesses to pay for extra capacity in the upstream auction decrease with the dominant firm F_0's initial capacity and therefore the upstream auction may be biased either in favor or against the dominant firm, depending on the degree of differentiation between products.

Other Allocations on the Downstream Market. Assume that firms' quantities are chosen so as to maximize a weighted sum of the advertiser expected surplus and the firms' profits and denote by $\alpha \in [0,1]$ the weight on firms' profits. An increase in α captures the idea that the bargaining outcome on the downstream market might be more favorable to the firms. The characterization of optimal allocations is straightforward and replicates the one we performed in Section 3.1, the sole change being that the virtual marginal cost now writes as $\tilde{\theta}_i = \theta_i + (1-\alpha)G(\theta_i)/g(\theta_i)$. In the Appendix, we show that as α increases, the set of parameters values such that the upstream auction is biased towards the dominant firm increases. The intuition is that, as α increases, the advertiser loses some of his monopsony power and his ability to screen the firms' marginal costs diminishes. The downstream allocation is tilted towards efficiency, firms produce more and get a greater share of the overall surplus. As more production is called for, the dominant firm F_0 can fulfill the advertiser's needs more often when it enjoys some post-merger extra capacity and if it is the most efficient supplier, an effect that increases its profits when winning the upstream auction. At the same time, if the weak firm F_1 is more efficient, an increase in its capacity reduces the probability that F_0 is called for and thus its profit. Overall, the willingness to pay of the dominant firm F_0 in the upstream auction increases.

5. MERGER WITH A DOWNSTREAM COMPETITOR

The purpose of this section is to extend our analysis to situations in which the merger involves firms that are direct competitors on the downstream market. To this end, let us add to our model a third firm, denoted by F_2, with initial capacity K_2 and marginal cost θ_2 (drawn from the same distribution G on the same support $[\underline{\theta}, \overline{\theta}]$).

Straightforward adaptations of Section 3.1 allow us to characterize the outcomes of downstream competition in terms of the firms' virtual valuations and the corresponding firms' profits. Applying then the methodology of Section 3.2 allows to show that optimal quantity profiles are given by

$$q_i(\theta_i,\theta_j,\theta_k,\varepsilon)=\begin{cases}\min(D(\tilde{\theta}_i)+\varepsilon;\hat{K}_i) & \text{if } \theta_i<\min_{k\neq i}(\theta_k)\ ,\\ \min(\max(D(\tilde{\theta}_i)+\varepsilon-\hat{K}_k;0);\hat{K}_i) & \text{if } \theta_j>\theta_i>\theta_k,\\ \min(\max(D(\tilde{\theta}_i)+\varepsilon-\hat{K}_j;0);\hat{K}_i) & \text{if } \theta_k>\theta_i>\theta_j,\\ \min(\max(D(\tilde{\theta}_i)+\varepsilon-(\hat{K}_j+\hat{K}_k);0);\hat{K}_i) & \text{if } \theta_i>\max_{k\neq i}(\theta_k),\end{cases}$$

where $\hat{K}_i$ denotes firm F_i's capacity after the upstream competition stage. The intuition closely follows that of the two-firm case. When a firm is the most efficient, it supplies the advertiser up to its capacity. Otherwise, it supplies the advertiser's residual demand, still up to its capacity. The difference is that the advertiser's residual demand depends on how many firms have already supplied the advertiser.

Then, we can derive F_i's expected profit

$$\begin{aligned}&\hat{V}_i(\hat{K}_i,\hat{K}_j,\hat{K}_k)=\\ &\quad\mathbb{E}_{\theta_i}\Bigg(\frac{G(\theta_i)}{g(\theta_i)}\Bigg((1-G(\theta_i))^2\int_0^{\overline{\varepsilon}}\min(D(\tilde{\theta}_i)+\varepsilon;\hat{K}_i)f(\varepsilon)d\varepsilon\\ &\qquad+G(\theta_i)(1-G(\theta_i))\int_{\hat{K}_k-D(\tilde{\theta}_i)}^{\overline{\varepsilon}}\min(D(\tilde{\theta}_i)+\varepsilon-\hat{K}_k;\hat{K}_i)f(\varepsilon)d\varepsilon\\ &\qquad+G(\theta_i)(1-G(\theta_i))\int_{\hat{K}_j-D(\tilde{\theta}_i)}^{\overline{\varepsilon}}\min(D(\tilde{\theta}_i)+\varepsilon-\hat{K}_j;\hat{K}_i)f(\varepsilon)d\varepsilon\\ &\qquad+G(\theta_i)^2\int_{\hat{K}_j+\hat{K}_k-D(\tilde{\theta}_i)}^{\overline{\varepsilon}}\min(D(\tilde{\theta}_i)+\varepsilon-(\hat{K}_j+\hat{K}_k);\hat{K}_i)f(\varepsilon)d\varepsilon\Bigg)\Bigg).\end{aligned}\tag{5.1}$$

Consider now the upstream market. A new issue is that the presence of negative externalities with more than two participants to the upstream auction may create intricate strategic effects with respect to the willingness of those firms to bid in the upstream auctions.[10] To simplify the analysis, and to focus on the most relevant scenario for the real world antitrust case that served as a motivation for our analysis, we limit ourselves to the case where only F_0 and F_1 participate to the upstream auction for additional capacity.

Before the merger, the willingness to pay of F_0 (resp. F_1) for extra capacity on the upstream market is given by $\Delta\hat{V}_0\equiv\hat{V}_0(K_0+k,K_1,K_2)-\hat{V}_0(K_0,K_1+k,K_2)$ (resp. $\Delta\hat{V}_1\equiv\hat{V}_1(K_0,K_1+k,K_2)-\hat{V}_1(K_0+k,K_1,K_2)$). Before the merger, the dominant firm F_0 wins when

$$\Delta\hat{V}_0>\Delta\hat{V}_1.$$

After the merger, the willingness to pay of F_0 (resp. F_1) becomes $\Delta V_0\equiv V_0(K_0+K_2+k,K_1)-V_0(K_0+K_2,K_1+k)$ (resp. $\Delta V_1\equiv V_1(K_0+K_2,K_1+k)-V_1(K_0+K_2+k,K_1)$), where the profit functions $V_i(\cdot)$ $(i=0,1)$ have been derived in Section 3 (see Equation (3.7)). After the merger, the dominant firm F_0 wins the upstream auction when

$$\Delta V_0>\Delta V_1.$$

[10]This issue is studied in depth in Jéhiel and Moldovanu (1996) and is outside the scope of this paper.

Observe now that $K_2 = 0$ implies $\hat{V}_i(K_i, K_j, 0) = V_i(K_i, K_j)$ and, thus, $(\Delta\hat{V}_0 - \Delta\hat{V}_1)|_{K_2=0} = (\Delta V_0 - \Delta V_1)|_{K_2=0}$. Thus, to understand how the merger impacts the upstream market, it suffices to understand how F_2's capacity K_2 impacts the firms' willingnesses to pay.

After the merger, the dominant firm's initial capacity is $K_0 + K_2$ and that of the weak firm is K_1. Therefore, we can use Proposition 4 and Condition (4.5) to characterize the impact of a change in K_2 on the firms' willingnesses to pay in the upstream auction. Note, in particular, that neither the nature nor the intensity of the bias in the upstream auction depend on the weak firm F_1's capacity K_1. Intuitively, this arises because, from the viewpoint of firms' willingnesses to pay, an increase in K_2 affects, first, the dominant firm through its ability to supply the advertiser when it is the most efficient firm and capacity-constrained, and, second, the weak firm through the residual demand that it supplies when it is the least efficient firm. Both impacts do not depend on K_1

Things are different before the merger, when three firms compete to supply the advertiser. First, when K_2 increases, this does not change a firm's ability to supply the advertiser when it is the most efficient firm and capacity-constrained. Such a change impacts the residual demand when a firm is the least or second least efficient firm in the market. Therefore, the impact of a change in K_2 on F_0's and F_1's willingnesses to pay now depends on all the initial capacity held by the three firms in the market.

We are now ready to state the next result.

Proposition 5. *Suppose K_2 is small. When the dominant firm F_0 merges with firm F_2, the upstream auction is biased towards the merged entity if and only if*

(5.2)
$$-\mathbb{E}_\theta\Bigg(\frac{G(\theta)(1-2G(\theta))}{g(\theta)}\Bigg((1-G(\theta))\Big(F(K_0+K_2+k-D(\tilde{\theta}))-F(K_0+K_2-D(\tilde{\theta}))\Big)$$
$$+G(\theta)\Big(F(K_1+k+K_2-D(\tilde{\theta}))-F(K_1+K_2-D(\tilde{\theta}))\Big)\Bigg)\Bigg)>0.$$

Condition (5.2) has some similarities with Condition (4.5). When the density of the demand shock is flat, only the distribution of marginal costs matters. When the density of demand shocks puts small weights on large shocks and large weights on small shocks, Condition (5.2) is likely to be violated. There are some interesting differences, though, which are best illustrated using our running example.

Running Example. Condition (5.2) now amounts to[11]

$$\frac{b\lambda\big(-2+(2-b\lambda)e^{2b\lambda}-b\lambda(3+2b\lambda)\big)}{-6+e^{2b\lambda}(6-4b\lambda+(b\lambda)^2)-b\lambda(8+5b\lambda+2(b\lambda)^2)} > -\frac{1-e^{-\lambda(K_0-K_1)}}{1+e^{-\lambda(K_0-K_1)}}. \tag{5.3}$$

This allows to obtain the following result.

Lemma 4. *There exists a threshold $\tilde{b\lambda}$, with $\tilde{b\lambda} > \underline{b\lambda}$ (where $\underline{b\lambda}$ is defined in Lemma 1), such that:*

- *if $b\lambda \leq \underline{b\lambda}$, then the upstream auction is biased towards the merged firm;*
- *if $b\lambda \geq \tilde{b\lambda}$, , then the upstream auction is biased towards the competitor of the merged firm;*

[11]Observe that this condition does not depend on K_2.

- *if* $b\lambda \in [\underline{b\lambda}, \tilde{b\lambda}]$*, then the upstream auction is biased towards the merged entity when the asymmetry between firms' initial capacity levels* $K_0 - K_1$ *is large enough.*

The logic is quite similar to the one unveiled in Proposition 4. When the demand on the downstream market is inelastic (b small), or when the distribution of the demand shock puts enough weight on high shocks (λ small), the upstream auction is biased towards the merged entity.

The new feature is that, for intermediate values of parameter $b\lambda$, the nature of the bias depends on F_0's and F_1's pre-merger capacities. As firms are more asymmetric in terms of their initial capacity levels, the upstream auction gets more biased towards the merged firm. Put differently, for $b\lambda \in [\underline{b\lambda}, \tilde{b\lambda}]$, there is a link between the competitive concerns raised by the merger on the upstream market and the degree of asymmetry between firms before the merger.

Interestingly, and in the same spirit as Section 4, the merger does not change the identity of the firm that wins the upstream auction for additional capacity.

6. Conclusion

In this paper, we presented a model whereby two firms differing in their initial capacity endowments compete on two fronts: first, on an upstream auction to acquire some extra capacities; and second, on a downstream market where those firms supply an advertiser with random demand. Due to the interaction between the upstream and the downstream markets, the auction for extra capacity on the upstream market entails externalities, as the willingness to pay of either firm depends on the difference between downstream profit when it wins the extra capacity and is better able to absorb the possible advertiser's large needs, and its profit when it loses and its rival is more often able to supply such needs. These willingnesses to pay for extra capacity therefore depend on the degree of ex ante asymmetry in their initial capacity endowments and on the overall magnitude of those endowments. In particular, the willingnesses to pay of both the dominant and the weak firms decrease with the dominant firm's initial capacity. This means that, in general, whether the upstream auction is biased towards the dominant firm, following a merger in which this firm acquires more capacity, is ambiguous. Nevertheless, using a particular specification of the model (uniform distribution for the firms' marginal costs and exponential distribution for demand shocks), we show that whether the dominant firm or the weak firm wins the upstream auction may sometimes entirely depend on the characteristics of the downstream market and not on the magnitude of the dominant firm's capacity. It is thus implied that, under such circumstances, a merger that increases the initial capacity of the dominant firm does not change the identity of the winning firm in the upstream auction. It therefore appears that there exist, unfortunately, no simple rules of thumb to guide competition authorities in the assessment of the impact of the merger on the upstream market, and a case-by-case approach is required.

As far as welfare is concerned, we show that the advertiser surplus on the downstream market increases with the total initial capacity of firms, but decreases with the asymmetry between firms with respect to their capacity levels. In our particular specification, we show that the net effect of a merger that increases the dominant firm's initial capacity is to increase the advertiser expected surplus.

REFERENCES

Allen B., R. Deneckere, T. Faith and D. Kovenock (2000). "Capacity Precommitment as a Barrier to Entry: A Bertrand-Edgeworth Approach," *Economic Theory*, 15: 501-530.

Burguet, R. and J. Sákovics (2017a). "Competitive Foreclosure," *The RAND Journal of Economics*, 48: 906-926.

Burguet, R. and J. Sákovics (2017b). "Bertrand and the Long Run," *International Journal of Industrial Organization*, 51: 39-55.

Davidson, C. and R. Deneckere (1986). "Long-run Competition in Capacity, Short-Run Competition in Price and the Cournot Model," *The RAND Journal of Economics*, 17: 404-415.

Deneckere, R. and D. Kovenock (1992). "Price Leadership," *The Review of Economic Studies,* 59: 143-162.

Deneckere, R. and D. Kovenock (1996). "Bertrand-Edgeworth Duopoly with Unit Cost Asymmetry," *Economic Theory*, 8: 1-25.

Eső, P., V. Nocke and L. White (2010). "Competition for Scarce Resources," *The RAND Journal of Economics*, 41: 524-548.

Fingleton, J. (1997). "Competition Among Middlemen When Buyers and Sellers Can Trade Directly," *The Journal of Industrial Economics*, 45: 405-427.

Fudenberg, D. and J. Tirole (1984). "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look," *The American Economic Review,* 74: 361-366.

Ghemawat, P. (1990). "The Snowball Effect," *International Journal of Industrial Organization*, 8: 335-351.

Jéhiel, P. and B. Moldovanu (1996). "Strategic Nonparticipation," *The RAND Journal of Economics*, 27: 84-98.

Jéhiel, P. and B. Moldovanu (2000). "Auctions with Downstream Interaction Among Buyers," *The RAND Journal of Economics,* 31: 768-791.

Jéhiel, P., B. Moldovanu and E. Stacchetti (1999). "Multidimensional Mechanism Design for Auctions with Externalities," *Journal of Economic Theory,* 85: 258-293.

Kreps, D. and J. Scheinkman (1983). "Quantity Precommitment and Bertrand Competition Yields Cournot Outcomes," *Bell Journal of Economics*, 14: 326-337.

Laffont, J.J. and D. Martimort (2002). *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press.

McAfee, P. (1998). "Four Issues in Market Design," *Revista de Análisis Económico*, 13: 7-24.

Mayo, J. and D. Sappington (2016). "When Do Auctions Ensure the Welfare-maximizing Allocation of Scarce Inputs?," *The RAND Journal of Economics*, 47: 186-206.

Myerson, R. (1981). "Optimal Auction Design," *Mathematics of Operations Research,* 6: 58-73.

Myerson, R. (1982). "Optimal Coordination Mechanisms in Generalized Principal-Agent Models," *Journal of Mathematical Economics,* 10: 67-81.

Myerson, R. and M. Satterthwaite (1983). “Efficient Mechanisms for Bilateral Trading,” *Journal of Economic Theory,* 29: 265-281.

Osborne, M. and C. Pitchick (1986). “Price Competition in a Capacity Constrained Duopoly,” *Journal of Economic Theory,* 38: 238-260.

Rey, P. and J. Tirole (2007). “A Primer on Foreclosure,” *Handbook of Industrial Organization*, Volume 3, eds. M. Armstrong and R. Porter, North-Holland.

Stahl, D.O. (1988). “Bertrand Competition for Inputs and Walrasian Outcomes,” *The American Economic Review*, 78: 189-201.

Yanelle, M.-O. (1997). “Banking Competition and Market Efficiency,” *The Review of Economic Studies*, 64: 215-239.

APPENDIX

PROOF OF PROPOSITION 1. Given the expected profits given by (3.5), quantities bought by the advertiser are solution of

$$\max_{(q_0(\theta_0,\theta_1,\varepsilon),q_1(\theta_1,\theta_0,\varepsilon))} \mathbb{E}_{(\theta_i,\theta_{-i},\varepsilon)}\left(S\big(\sum_{i=0,1} q_i(\theta_i,\theta_{-i},\varepsilon)-\varepsilon\big)-\sum_{i=0,1}\tilde{\theta}_i q_i(\theta_i,\theta_{-i},\varepsilon)\right),$$
$$\text{subject to } q_i(\theta_i,\theta_{-i},\varepsilon)\le \hat{K}_i \quad \forall\varepsilon, \forall i\in\{0,1\}.$$

The objective can be maximized pointwise so that quantities are solution of

$$\max_{(q_0(\theta_0,\theta_1,\varepsilon),q_1(\theta_1,\theta_0,\varepsilon))} S\big(\sum_{i=0,1} q_i(\theta_i,\theta_{-i},\varepsilon)-\varepsilon\big)-\sum_{i=0,1}\tilde{\theta}_i q_i(\theta_i,\theta_{-i},\varepsilon),$$
$$\text{subject to } q_i(\theta_i,\theta_{-i},\varepsilon)\le \hat{K}_i \quad \forall i\in\{0,1\}.$$

Suppose that firm F_i is more efficient than firm F_{-i}, that is, $\theta_i < \theta_{-i}$. Thanks to the monotone hazard rate property of distribution $G(\cdot)$, this implies that $\tilde{\theta}_i \le \tilde{\theta}_{-i}$. Since firm F_i is more efficient, the maximand is maximized when F_i produces a quantity such that $S'(q_i-\varepsilon)=\tilde{\theta}_i$, or $q_i = D(\tilde{\theta}_i)+\varepsilon$, and firm F_{-i} does not supply. Different cases have then to be considered to check whether capacity constraints are binding or not.

Suppose $\hat{K}_i \ge D(\tilde{\theta}_i)+\varepsilon$. Firm F_i can indeed supply such a quantity because its capacity is large enough. Suppose now $\hat{K}_i \le D(\tilde{\theta}_i)+\varepsilon$. Firm F_i supplies up to its capacity, that is, $q_i=\hat{K}_i$. The advertiser may then buy a positive quantity from the least efficient firm F_{-i}. How much quantity depends on F_{-i}'s capacity. Production of the most efficient firm F_i is therefore given by $\min(D(\tilde{\theta}_i)+\varepsilon;\hat{K}_i)$.

Consider the least efficient firm F_{-i}, which produces only when F_i is capacity-constrained. Given that a quantity $\hat{K}_i$ has already been bought to firm F_i, $q_{-i}(\theta_i,\theta_{-i},\varepsilon)$ is solution of

$$\max_{q_{-i}(\theta_{-i},\theta_i,\varepsilon)} S\big(\hat{K}_i+q_{-i}(\theta_{-i},\theta_i,\varepsilon)-\varepsilon\big)-\tilde{\theta}_{-i}q_{-i}(\theta_{-i},\theta_i,\varepsilon),$$
$$\text{subject to } q_{-i}(\theta_{-i},\theta_i,\varepsilon)\le \hat{K}_{-i}.$$

The maximand is maximized when $q_{-i}(\theta_{-i},\theta_i,\varepsilon)=D(\tilde{\theta}_{-i})+\varepsilon-\hat{K}_i$ provided this quantity is positive. Therefore, if $\hat{K}_i \ge D(\tilde{\theta}_{-i})+\varepsilon$, then $q_{-i}=0$. Suppose now that $\hat{K}_i \le D(\tilde{\theta}_{-i})+\varepsilon$. If $\hat{K}_{-i} \ge D(\tilde{\theta}_{-i})+\varepsilon-\hat{K}_i$, then F_{-i} can supply all the advertiser's residual demand and $q_{-i}=D(\tilde{\theta}_{-i})+\varepsilon-\hat{K}_i$. Otherwise, that is, when $\hat{K}_{-i}\le D(\tilde{\theta}_{-i})+\varepsilon-\hat{K}_i$, firm F_{-i} sells its capacity $\hat{K}_{-i}$. Production of the least efficient firm writes thus as $\min(\max(D(\tilde{\theta}_{-i})+\varepsilon-\hat{K}_i;0);\hat{K}_{-i})$.

PROOF OF PROPOSITION 2. Differentiating (3.7) with respect to K_i and K_{-i} leads to

$$\text{(A.1)}\quad \frac{\partial V_i}{\partial K_i}(K_i, K_{-i}) = \mathbb{E}_{\theta_i}\left(\frac{G(\theta_i)}{g(\theta_i)}\Big((1-G(\theta_i))(1-F(K_i - D(\tilde{\theta}_i))) + G(\theta_i)(1-F(K_i+K_{-i}-D(\tilde{\theta}_i)))\Big)\right) > 0;$$

$$\text{(A.2)}\quad \frac{\partial^2 V_i}{\partial K_i^2}(K_i, K_{-i}) = \mathbb{E}_{\theta_i}\left(\frac{G(\theta_i)}{g(\theta_i)}\Big(-(1-G(\theta_i))f(K_i - D(\tilde{\theta}_i)) - G(\theta_i)f(K_i+K_{-i}-D(\tilde{\theta}_i))\Big)\right) < 0;$$

$$\text{(A.3)}\quad \frac{\partial V_i}{\partial K_{-i}}(K_i, K_{-i}) = \mathbb{E}_{\theta_i}\left(\frac{G^2(\theta_i)}{g(\theta_i)}\Big(F(K_{-i} - D(\tilde{\theta}_i)) - F(K_i+K_{-i}-D(\tilde{\theta}_i))\Big)\right) < 0.$$

Expressions (A.1), (A.2) and (A.3) prove Proposition 2.

PROOF OF PROPOSITION 3. Using (A.1) twice, first expressed at (K_0+k, K_1) and then at (K_0, K_1+k), we obtain the first item in Proposition 3. Using (A.3) twice, first expressed at (K_1+k, K_0) and then at (K_1, K_0+k), we obtain the second item in Proposition 3.

PROOF OF PROPOSITION 4. Observe that when $K_0 = K_1$, firms have identical willingnesses to pay in the upstream auction. Then, combining (4.3) and (4.4) leads to (4.5).

PROOF OF LEMMA 1. Computations show that

$$(V_0(K_0+k, K_1) - V_0(K_0, K_1+k)) - (V_1(K_0, K_1+k) - V_1(K_0+k, K_1)) = -\frac{e^{(a-2b-k-K_0-K_1)\lambda}(e^{k\lambda}-1)(e^{K_0\lambda}-e^{K_1\lambda})}{4b^3\lambda^4}\left(e^{2b\lambda}(-2+b\lambda)+2+3b\lambda+2(b\lambda)^2\right).$$

Therefore,

$$\frac{\partial}{\partial K_0}(V_0(K_0+k, K_1) - V_0(K_0, K_1+k)) - (V_1(K_0, K_1+k) - V_1(K_0+k, K_1)) = -\frac{e^{(a-2b-k-K_0)\lambda}(e^{k\lambda}-1)}{4b^3\lambda^3}\left(e^{2b\lambda}(-2+b\lambda)+2+3b\lambda+2(b\lambda)^2\right).$$

Condition (4.5) rewrites thus as

$$-\left(e^{2b\lambda}(-2+b\lambda)+2+3b\lambda+2(b\lambda)^2\right) > 0.$$

We can then show that the function $x \mapsto -(e^{2x}(-2+x)+2+3x+2x^2)$ is strictly positive for $x \in (0, \underline{x})$ and strictly negative for $x > \underline{x}$, where $\underline{x} \approx 1.344$.

PROOF OF LEMMA 2. The expected surplus of the representative advertiser on the downstream market can be written as

$$\mathbb{E}_{(\theta_0,\theta_1,\varepsilon)}\left(S\big(\sum_{i=0,1} q_i(\theta_i,\theta_{-i},\varepsilon)-\varepsilon\big) - \sum_{i=0,1}\theta_i q_i(\theta_i,\theta_{-i},\varepsilon) - \sum_{i=0,1} U_i(\theta_i)\right),$$

where quantity $q_i(\theta_0,\theta_1,\varepsilon)$ is given by Proposition 1 and profit $U_i(\theta_i)$ is given by Equation (3.6). The surplus corresponding to a demand $D(p) = a+\varepsilon-bp$ is $S(q-\varepsilon) = (a/b-(q-\varepsilon)/(2b))(q-\varepsilon)$. Let $CS(\hat{K}_0, \hat{K}_1)$ denote the expected advertiser surplus when firms have capacities $(\hat{K}_0, \hat{K}_1)$.

Since expected profits are given by $V_i(\hat{K}_i, \hat{K}_{-i}) = \mathbb{E}_{\theta_i}(U_i(\theta_i))$ from Equation (3.7), it suffices to compute the expected welfare

$$\mathbb{E}_{(\theta_0,\theta_1,\varepsilon)}\Big(S\big(\sum_{i=0,1} q_i(\theta_i,\theta_{-i},\varepsilon) - \varepsilon\big) - \sum_{i=0,1} \theta_i q_i(\theta_i,\theta_{-i},\varepsilon)\Big) = \int_{\underline{\theta}}^{\overline{\theta}} \Big[\int_{\underline{\theta}}^{\theta_1} I_0 dG(\theta_0) + \int_{\theta_1}^{\overline{\theta}} I_1 dG(\theta_0)\Big] dG(\theta_1),$$

where

$$\begin{aligned} I_0 = &\int_0^{\hat{K}_0 - D(\tilde{\theta}_0)} \big(S(D(\tilde{\theta}_0)) - \theta_0(D(\tilde{\theta}_0)+\varepsilon)\big) dF(\varepsilon) \\ &+ \int_{\hat{K}_0-D(\tilde{\theta}_0)}^{\hat{K}_0-D(\tilde{\theta}_1)} \big(S(\hat{K}_0-\varepsilon) - \theta_0(\hat{K}_0)\big) dF(\varepsilon) \\ &+ \int_{\hat{K}_0-D(\tilde{\theta}_1)}^{\hat{K}_0+\hat{K}_1-D(\tilde{\theta}_1)} \big(S(D(\tilde{\theta}_1)) - \theta_0(\hat{K}_0) - \theta_1(D(\tilde{\theta}_1)+\varepsilon-\hat{K}_0)\big) dF(\varepsilon) \\ &+ \int_{\hat{K}_0+\hat{K}_1-D(\tilde{\theta}_1)}^{+\infty} \big(S(\hat{K}_0+\hat{K}_1-\varepsilon) - \theta_0(\hat{K}_0) - \theta_1(\hat{K}_1)\big) dF(\varepsilon), \end{aligned}$$

and

$$\begin{aligned} I_1 = &\int_0^{\hat{K}_1 - D(\tilde{\theta}_1)} \big(S(D(\tilde{\theta}_1)) - \theta_1(D(\tilde{\theta}_1)+\varepsilon)\big) dF(\varepsilon) \\ &+ \int_{\hat{K}_1-D(\tilde{\theta}_1)}^{\hat{K}_1-D(\tilde{\theta}_0)} \big(S(\hat{K}_1-\varepsilon) - \theta_1(\hat{K}_1)\big) dF(\varepsilon) \\ &+ \int_{\hat{K}_1-D(\tilde{\theta}_0)}^{\hat{K}_0+\hat{K}_1-D(\tilde{\theta}_0)} \big(S(D(\tilde{\theta}_0)) - \theta_1(\hat{K}_1) - \theta_0(D(\tilde{\theta}_0)+\varepsilon-\hat{K}_1)\big) dF(\varepsilon) \\ &+ \int_{\hat{K}_0+\hat{K}_1-D(\tilde{\theta}_0)}^{+\infty} \big(S(\hat{K}_0+\hat{K}_1-\varepsilon) - \theta_1(\hat{K}_1) - \theta_0(\hat{K}_0)\big) dF(\varepsilon). \end{aligned}$$

Last, let $\Delta K \equiv (\hat{K}_0 - \hat{K}_1)/2 \geq 0$ and $K \equiv (\hat{K}_0 + \hat{K}_1)/2$. We thus have $\hat{K}_0 = K + \Delta K$ and $\hat{K}_1 = K - \Delta K$.

Computations show that

$$\frac{\partial}{\partial \Delta K} CS(K+\Delta K, K-\Delta K) = -\frac{e^{\lambda(a-2b-(K+\Delta K))}(e^{2\Delta K\lambda}-1)}{2b^3\lambda^3}\left(e^{2b\lambda}(-1+b\lambda)+1+b\lambda\right),$$

which is strictly negative for $b\lambda > 0$. Therefore, the expected surplus is decreasing in ΔK.

Computations also show the expected surplus increases with average capacity (that is, $\frac{\partial CS}{\partial K}(K+\Delta K, K-\Delta K) \geq 0$) iff

$$f(x,y) = -2 + e^{2x}\big(2 + y(-1+x)\big) - 4x + y(1+x) \geq 0,$$

where $y = e^{\lambda K_0} + e^{\lambda K_1}$ and $x = b\lambda$. Note that $f(x,0) \geq 0$ for all x and $\partial f/\partial y(x,y) \geq 0$. Therefore, welfare is increasing in K.

Last, $|\frac{\partial}{\partial \Delta K} CS| \leq \frac{\partial CS}{\partial K}$ amounts to $e^{\lambda \Delta K}(-1-2x+e^{2x}) + e^{\lambda K}(1+x+e^{2x}(-1+x)) \geq 0$, which always holds.

PROOF OF LEMMA 3. The difference between the advertiser surplus when firm F_1 wins the upstream auction and when firm F_0 wins is given by

$$CS(K_0, K_1 + k) - CS(K_0 + k, K_1) = \frac{(e^{\lambda k} - 1)(e^{\lambda K_0} - e^{\lambda K_1})e^{(a-2b-k-K_0-K_1)\lambda}}{2b^3\lambda^4}\left(1 + b\lambda + e^{2b\lambda}(b\lambda - 1)\right),$$

which is always positive.

Moreover, computations show that

$$\frac{\partial}{\partial K_0} CS(K_0 + k, K_1) = \frac{e^{(a-2b-k-K_0-K_1)\lambda}}{2b^3\lambda^3}\left(-1 + e^{2b\lambda} - 2b\lambda + e^{(2b+K_1)\lambda}(-1 + b\lambda) + e^{K_1\lambda}(1 + b\lambda)\right) \geq 0.$$

The expression in parenthesis is strictly increasing in K_1 and strictly positive for $K_1 = 0$. Therefore, $CS(K_0 + k, K_1)$ increases with K_0.

One can show in a similar way that $CS(K_0, K_1 + k)$ also increases with K_0.

COURNOT COMPETITION WITH DIFFERENTIATED PRODUCTS AND CAPACITY CONSTRAINTS. Suppose that firms compete in quantities in the downstream market. The inverse demand faced by firm F_i is given by $P_i(q_i, q_{-i}) = a - q_i - \gamma q_{-i} + \varepsilon$, with $\gamma \in [0, 1]$ being the degree of product substitutability. Suppose both firms have the same marginal cost ($\theta_0 = \theta_1 = \theta$) and that $K_0 \geq K_1 + k$. The game now unfolds as follows.

- Firms compete in the upstream auction for the k extra units.

- The common marginal cost and the demand shock realize and are common knowledge.

- Firms compete in quantities to supply the downstream market given their capacity levels.

Given some capacities $(\hat{K}_0, \hat{K}_1)$, with $\hat{K}_0 \geq \hat{K}_1$, three configurations are possible depending on the realization of the demand shock: (i) none of the firms is capacity-constrained (which amounts to $\varepsilon \leq \hat{K}_1(2 + \gamma) - a + \theta$), and both firms play the standard Cournot best-response; (ii) both firms are capacity-constrained (which amounts to $\varepsilon \geq \hat{K}_0(2 + \gamma) - a + \theta$), and both firms produce at capacity; (iii) the weak firm F_1 is capacity-constrained whereas the dominant firm F_0 is not (which amounts to $\hat{K}_1(2 + \gamma) - a + \theta \leq \varepsilon \leq \hat{K}_0(2 + \gamma) - a + \theta$), and F_1 plays $\hat{K}_1$ whereas F_0 play the Cournot best-response.

Given these quantities produced on the downstream market, we can compute the firms' profits in expectation over the demand shock and the marginal cost. By analogy with our main analysis, denote those profits by $V_0^C(\hat{K}_0, \hat{K}_1)$ and $V_1^C(\hat{K}_0, \hat{K}_1)$.

One can then show that

$$\frac{\partial}{\partial K_0}\left(V_0^C(K_0 + k, K_1) - V_0^C(K_0, K_1 + k)\right) = \frac{-1}{\lambda^2} e^{\lambda(a-1-2(k+K_0)-\gamma(k+K_1))}(e^{\lambda} - 1)(e^{2k\lambda} - e^{k\gamma\lambda}),$$

$$\frac{\partial}{\partial K_0}\left(V_1^C(K_0, K_1 + k) - V_1^C(K_0 + k, K_1)\right) = \frac{-1}{\lambda} e^{\lambda(a-1-2K_0-K_1\gamma-(2+\gamma)k)}(e^{\lambda} - 1)(e^{2k\lambda}(k + K_1) - e^{k\gamma\lambda}K_1),$$

which are both negative for $\gamma \in [0, 1]$. Hence, willingnesses to pay are decreasing with K_0.

The difference between both willingnesses to pay varies with K_0 as follows

$$\frac{\partial}{\partial K_0}\Big((V_0^C(K_0+k,K_1)-V_0^C(K_0,K_1+k))-(V_1^C(K_0,K_1+k)-V_1^C(K_0+k,K_1))\Big)=$$
$$\frac{1}{\lambda^2}e^{\lambda(a-1-2K_0-K_1\gamma-(2+\gamma)k)}(e^{\lambda}-1)\left(e^{k\gamma\lambda}(1-K_1\gamma\lambda)+e^{2k\lambda}(-1+(k+K_1)\gamma\lambda)\right).$$

This expression is strictly negative for $\gamma=0$ and strictly positive for $\gamma=1$.

ALTERNATIVE SELECTION OF DOWNSTREAM OUTCOMES. Suppose now that downstream quantities are chosen so as to maximize a weighted sum of the advertiser surplus and firms' profits (omitting some arguments)

$$\max_{(q_0,q_1)}\mathbb{E}\Big(S\big(\sum_{i=0,1}q_i-\varepsilon\big)-\sum_{i=0,1}\theta_iq_i-(1-\alpha)\sum_{i=0,1}U_i(\theta_i)\Big),$$

where $\alpha\in[0,1]$ is the weight on firms' profit. Our characterization of downstream outcomes can be immediately adapted, the sole change being that the virtual marginal cost of firm F_i now writes as $\tilde{\theta}_i=\theta_i+(1-\alpha)G(\theta_i)/g(\theta_i)$. Equation (4.6) now becomes (up to some multiplicative terms)

$$4-(2-\alpha)b\lambda-e^{-(2-\alpha)b\lambda}(4+3(2-\alpha)b\lambda+((2-\alpha)b\lambda)^2)>0.$$

Essentially, the term $b\lambda$ has been replaced by $(2-\alpha)b\lambda$. Therefore, as α increases, quantities increase, and the auction is more biased towards the dominant firm (the set of parameter values for which the auction is biased towards the dominant firm is larger).

PROOF OF PROPOSITION 5. Expressions (A.1) and (A.3) lead to

$$\frac{\partial}{\partial K_2}(\Delta V_0-\Delta V_1)=\mathbb{E}_\theta\Big(\frac{G(\theta)}{g(\theta)}\Big(-(1-2G(\theta))(F(K_0+K_2+k-D(\tilde{\theta}))-F(K_0+K_2-D(\tilde{\theta})))\Big).$$

Next, differentiating Equation (5.1), we obtain (keeping in mind that K_2 small implies that $K_2-D(\tilde{\theta}_2)<0$ for all $\tilde{\theta}_2$)

$$\frac{\partial\hat{V}_i}{\partial K_j}(K_i,K_j,K_k)=$$
$$\mathbb{E}_{\theta_i}\Big(\frac{G(\theta_i)}{g(\theta_i)}\Big(-G(\theta_i)(1-G(\theta_i))F(K_i+K_j-D(\tilde{\theta}_i))$$
$$-G^2(\theta_i)(F(K_i+K_j+K_k-D(\tilde{\theta}_i))-F(K_j+K_k-D(\tilde{\theta}_i)))\Big)\Big).$$

Using this expression, we obtain

$$\frac{\partial}{\partial K_2}\Delta\hat{V}_0=$$
$$\mathbb{E}_\theta\Big(\frac{G(\theta)}{g(\theta)}\Big(-G(\theta)(1-G(\theta))(F(K_0+k+K_2-D(\tilde{\theta}))-F(K_0+K_2-D(\tilde{\theta})))$$
$$-G^2(\theta)(F(K_1+k+K_2-D(\tilde{\theta}))-F(K_1+K_2-D(\tilde{\theta})))\Big)$$

and

$$\frac{\partial}{\partial K_2}\Delta\hat{V}_1 = \mathbb{E}_\theta\left(\frac{G(\theta)}{g(\theta)}\Big(- G(\theta)(1-G(\theta))(F(K_1+k+K_2-D(\tilde{\theta}))-F(K_1+K_2-D(\tilde{\theta}))) - G^2(\theta)(F(K_0+k+K_2-D(\tilde{\theta}))-F(K_0+K_2-D(\tilde{\theta})))\Big)\right).$$

This finally leads to

$$\frac{\partial}{\partial K_2}\left((\Delta V_0-\Delta V_1)-(\Delta\hat{V}_0-\Delta\hat{V}_1)\right) = -\mathbb{E}_\theta\left(\frac{G(\theta)(1-2G(\theta))}{g(\theta)}\left((1-G(\theta))\Big(F(K_0+K_2+k-D(\tilde{\theta}))-F(K_0+K_2-D(\tilde{\theta}))\Big) + G(\theta)\Big(F(K_1+k+K_2-D(\tilde{\theta}))-F(K_1+K_2-D(\tilde{\theta}))\Big)\right)\right).$$

PROOF OF LEMMA 4. The right-hand side in Condition (5.3) belongs to $[0,-1)$ for any $b\lambda \geq 0$ and $K_0 \geq K_1$, and is decreasing in the degree of asymmetry between firms $K_0 - K_1$. The left-hand side in Condition (5.3) is: positive for $b\lambda \in [0, \underline{b\lambda}]$; decreasing between 0 and -1 for $b\lambda \in [\underline{b\lambda}, \tilde{b\lambda}]$ where $\tilde{b\lambda} \approx 2.535$; strictly below -1 for $b\lambda \geq \tilde{b\lambda}$. Hence, when $b\lambda \leq \underline{b\lambda}$, Condition (5.3) is satisfied. For $b\lambda > \tilde{b\lambda}$, Condition (5.3) is never satisfied. For $b\lambda$ on $[\underline{b\lambda}, \tilde{b\lambda}]$, there exists a unique level of $K_0 - K_1$ such that Condition (5.3) is satisfied if and only if $K_0 - K_1$ is below that level.