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WORKING TIME AND WAGE RATE DIFFERENCES: A CONTRACT THEORY APPROACH

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Working time and wage rate differences:

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Abstract

In the labor economics literature, discrimination is often defined as occurring when identically productive workers, placed in the same working conditions, are assigned contracts involving, in particular, different hourly wage rates. This paper applies contract theory to explain how in some circumstances such differences take place, even if contract discrimination and productivity differences are strictly ruled out. It is assumed that worker types differ only in their consumption/leisure preferences and in their availability. A labor cost-minimizing firm offers a menu of labor contracts, and lets workers self-select. The model reveals external effects between types and the possibility of a paradoxical situation in which less demanding workers obtain a higher wage rate. A mixed employment regime always requires a minimum number (a quantum) of most demanding workers.

Keywords: Working hours, Wage gap, Labor market discrimination, Contract theory.

JEL Classification: D86; J31; J41; J71.

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1 Introduction

According to the standard labour market theory, wage rates tend to reflect the marginal productivity of labour services. Consequently, wage rates should coincide for workers performing the same task with the same level of ability. Labour market observations however, in many instances, tend to challenge this view. Despite convergence in educational attainments and other relevant explanatory variables, unexplained differences still exist between the full-time wages of men and women, particularly at the top of the wage distribution (Gunderson, 1989; Blau and Kahn, 2017; Cahuc et al., 2014; OECD, 2018; Neumark, 2018).

To explain the differences between the neoclassical expected equality and observed facts, economists have often resorted to the notion of discrimination (Becker, 1957). Labor market discrimination has been defined as "a situation in which persons who provide labor market services and who are equally productive in a psychical or material sense are treated unequally in a way that is related to an observable characteristic such as race, ethnicity, or gender" (Altonji and Blank, 1999). It is well known, however, that some observed differences in wages obtained by equally competent workers may be explained without resorting to discrimination. The theory of compensating differentials, originating in Adam Smith's celebrated seminal works, argues that wage gaps can be grounded in differences in preferences for job attributes (Thaler and Rosen, 1975; Rosen, 1986).¹ Some other explanations resort to a productivity argument: in her presidential address to the 126th Annual Meeting of the American Economic Association, Goldin (2014) pointed out that in some qualified occupations (legal services, business, and finance), longer hours provided by men in general, are rewarded at a higher wage rate.² Goldin explains this outcome based on productivity increasing with hours, as firms and their clients value "temporal flexibility": on-site presence, intensive client contact, face-to-face time, etc. Workers who work longer hours and ac-

¹ As a recent illustration, Cook and al. (2018) study the wages earned by Uber drivers, and find a 7% gap favoring men. Since longer working schedules apparently have in this case little impact on the wage rate, it is suggested that male drivers earn a compensating differential for their willingness to drive in areas with higher crime rates and more drinking establishments. It must be noticed that in such a case, male and female labour services are not perfect substitutes.

² Cortés and Pan (2019) offer empirical evidence documenting a positive relationship between the demand for long hours and the wage gap for high-skilled employees, while Denning et al. (2019) find no such relationship. Mas and Pallais (2017) find from experimental data that women are relatively averse to employer imposed hour flexibility.

cept fragmented schedules could be more valuable to a firm since they can exchange information with their peers more efficiently. If men are more available than women in such jobs, then a gender wage gap could result.

This paper provides a model of wage and hours differentiation consistent with the above-mentioned observations that is completely free of productivity differences or discriminatory practices as defined by the labor market literature: in our model, all workers have equal access to all contracts proposed by the employer. The fundamental source of heterogeneity in the model is workers' disutility from work. For the sake of parsimony, we assume that workers are of only two types, and workers of the first type always demand less compensation for any amount of working time than workers of the second type. The other basic assumption of this model is that workers of the first type are in limited supply.

The principal seeks to hire workers for a predetermined number of working hours, as required to achieve a production target. The goal of the firm is to minimize the total cost of labor. If workers of both types were available in any quantity, cost minimization would generally cause labour demand to concentrate on the "less expensive" type. The more demanding workers being crowded out, they do not appear in the data. Our analysis focuses on the nontrivial, and most plausible situation, where less demanding workers are in scarce supply and the demand for hours is great enough to prompt the employer to hire both types of workers.

When contract discrimination is not possible, hiring workers of the more demanding type creates an externality influencing the contract offered to the less demanding type, through an incentive compatibility constraint that ensures the efficient self-selection of workers. The determination of optimal contracts builds on standard principles of contract theory (inter alia, Bolton and Dewatripont, 2005; Salanié, 2005; Laffont and Martimort, 2009), to which we add, as an original theoretical contribution, an endogenous determination of the proportion of worker types. Despite the relatively complex structure of the problem, the solution is fully characterized for a quadratic compensation function.

A compensation function defines the level of consumption required by a worker to fulfill his/her participation constraint. Our analysis reveals that ordering worker types by non-crossing compen-

sation functions is not sufficient to predict the ordering of optimal working times, compensations and implicit hourly wages. In particular, the model points out the crucial role played by the specific assumption concerning the sensibility of the worker compensation differential with respect to working time itself.

The analysis also reveals that the employer minimizes the total labour cost by offering contracts generally exhibiting different (implicit) hourly wages. We refer to these wage differences as forms of "pseudo-discrimination", since these wage rate differences between workers doing the same job with the same level of skills exist, but are not explained by incorrectly perceived productivity differences or by a biased objective function of the employer. In particular, the model reveals the possibility of a paradoxical situation in which less demanding workers are granted a higher wage rate, in sharp contrast to the case in which discrimination is possible.

Finally, in formalizing and interpreting the labour cost function, the model explains the demand for more exacting workers and predicts local discontinuity. For a threshold of needed working services, the cost minimizing policy switches from employing less demanding workers only, to the adoption of a mixed labour force, including a minimum number (quantum) of the more demanding type. This setting is compatible with some forms of type-specific mass redundancy in the case of an economic slowdown.

Our analysis can shed light on the topic of gender discrimination if consumption/leisure preferences are specific to the gender of the employee. Because women still have the responsibility of caring about their children in many countries, they might obtain more utility from out-of-job hours (Cain, 1986; OECD, 2018). Women can therefore be represented as the group of more demanding workers. Men would then represent the group of less demanding workers. If "less expensive" male labour is in short supply, the model shows possible gender wage differentials in the absence of any biased information about productivity or a nonconventional objective of the employer involving contract discrimination. As such, the tools used to study hours/wage offers are quite standard (contract theory); however, including the demanded hours constraint in the optimization problem and solving it explicitly with quadratic compensation functions can be seen as a contribution to this literature.

The paper is organized as follows: In section 2, we introduce our main assumptions and define the mixed employment regime. In sections 3, 4 and 4 the model is analyzed in three increasingly detailed sets of assumptions. Section 3 analyses the cost minimization problem for the general case. In section 4, we provide a more precise definition of optimal contracts when the compensation differential is a linearly increasing function of working hours. In section 5 more precision is achieved by adopting a quadratic compensation function, and numerical simulations are used to support the analytical results. The minimum number of type 2 employed workers is endogenous. Section 6 concludes.

2 Main assumptions

We analyze the *cost minimizing labor contracts* designed by an employer who needs a given amount of labour services H to achieve its production target.³ Technology is such that H can be expressed by a sum of hours considered as perfect substitutes in production. Working hours are homogeneous not only in terms of productivity but also in terms of working conditions, as applying to the same task. Differences related to the intrinsic disutility of work, as considered by the theory of compensating differentials, are therefore ruled out.

a/ Preferences

The utility function of a worker i is represented by $u_i = c_i - v_i(h)$, where c_i is consumption obtained from trading hours in the labor market, and $v_i(h)$ stands for the disutility of working h hours. In line with standard neoclassical assumptions, we assume that $v_i(0) = 0$, $v'_i(h) > 0$ and $v''_i(h) > 0$. The participation constraint of a worker is $u_i = c_i - v_i(h) \geq 0 \Leftrightarrow c_i \geq v_i(h)$. Therefore, $v_i(h)$ can be interpreted as the *compensation function*, indicating the *minimum* compensation required by an individual i to supply h hours of work.⁴

To keep the analysis simple, we assume that there are only two types of workers, $i = (1, 2)$, with compensation functions $v_1(h)$ and $v_2(h)$ respectively. The two compensation functions cross at the origin, for $h = 0$. With convex and increasing compensation functions, two cases can be

³ The cost minimization assumption is quite general, as any firm, be it a profit maximizer or not, should address it as a first-stage decision.

⁴ For a general form of the utility function, the compensation function would be defined by solving $U(c, h) = 0$ for an explicit $c = v(h)$. If $U(c, h)$ has standard neoclassical properties, then $v(h)$ is increasing in h and is convex.

considered:

1. The two compensation functions cross for some $h > 0$. This situation can occur, but it leads to trivial conclusions: there is one consumption/hours contract that is unambiguously preferred by each type of worker. The first-best solution prevails irrespective of worker availability.

2. The two compensation functions do not cross for $h > 0$. This assumption is tantamount to having type 2 workers always demanding *more* compensation than type 1 workers for any amount of working time:

$$v_2(h) > v_1(h), \forall h > 0. \quad (1)$$

The two types of workers are consequently unequivocally ordered in this respect. This case leads to nontrivial situations in which a nondiscriminating employer must take into account worker incentive compatibility constraints. If women receive a higher utility from out-of-employment hours, women might be considered representative of more demanding type 2 workers, with men being representative of less demanding type 1 workers.

Let us denote the *compensation premium* of type 2 workers by $\varsigma(h) = v_2(h) - v_1(h)$, with $\varsigma(h) > 0$. Letting aside other compensation profiles, we further assume that the premium $\varsigma(h)$ is monotonically increasing with h $\varsigma'(h) > 0, \forall h$ or:

$$\varsigma'(h) = v_2'(h) - v_1'(h) > 0, \quad (2)$$

b/ **Cost of labor**

The cost of labour includes worker compensation and a per individual, fixed, nonwage expense (cost of a work place unrelated to worked hours), denoted by $\theta > 0$ (Rosen, 1968; Hart, 1987; Contensou and Vranceanu, 2001). This fixed cost is assumed to be independent of the worker type.

We define \hat{h}_i as the *contract hours that minimizes the hourly cost of each type i regardless of any other constraint*:

$$\hat{h}_1 = \arg \min_h \left\{ \frac{v_1(h) + \theta}{h} \right\} \quad (3)$$

$$\hat{h}_2 = \arg \min_h \left\{ \frac{v_2(h) + \theta}{h} \right\} \quad (4)$$

We can refer to these hours as "hourly-cost minimizing" (HCM).

c/ Labor contracts

The employer can offer labor contracts defining hours of work and compensation (h, c) . We make the basic assumption that *the employer cannot discriminate*, i.e., cannot prevent workers from choosing their preferred contract. This can be interpreted as a consequence of either legal or social constraints banning discrimination when types can be observed (the case of gender for instance), or just of imperfect information with the employer not being able to distinguish the types. When contract discrimination is impossible, the principal must offer two labor contracts $P_1 = (h_1, c_1)$ and $P_2 = (h_2, c_2)$ and let workers self-select for their preferred one. The employer must keep in mind that hiring type 2 workers with contract P_2 introduces a costly constraint on contract P_1 , which must now be at least as attractive as P_2 for type 1 workers. This supplementary constraint is equivalent to a fixed cost that must be paid no matter how small is the number of type 2 workers hired.

d/ Availability of workers

Let n_1 be the number of type 1 workers and let n_2 be the number of type 2 workers hired by the firm. A second important assumption is that there is a *relative shortage* of type 1, less demanding workers. Let \bar{n}_1 be the number of type 1 workers available in the labour market. Type 2 workers are abundant, i.e., they are available in any number demanded by the firm.

e/ The employment regime

If total demand for hours is relatively low, there is no shortage of type 1 workers ($n_1 < \bar{n}_1$) and the employer would minimize the labor cost by employing only the less expensive type 1 workers. In this case the optimal contract is $P_1 = (\hat{h}_1, v_1(\hat{h}_1))$; it can be easily verified that the more demanding type 2 workers would not choose it. If all type 1 workers are employed ($n_1 = \bar{n}_1$), the employer might still want to work with only this type of workers. The employer might ask these workers to work longer hours, and compensate them more, in line with their convex compensation function. However, if the demand for hours is great enough, the high cost of the longer hours for a type 1 worker can justify the hiring of the more demanding type 2 workers. In the following, the analysis will focus on the most interesting case in which the demand for hours is great enough to

justify the employment of both types of workers, which we refer to as a *mixed-employment regime*.

We denote this threshold volume of hours by H_M .

3 The general problem

The goal of the firm is to minimize the cost of a total number of hours H to be provided by a limited number of type 1 workers and a flexible number of type 2 workers. Contract discrimination is not allowed. We follow the standard resolution steps applied in contract theory analysis (e.g., Laffont and Martimort, 2001); we first determine feasible allocations taking into consideration both participation and incentive compatibility constraints, and then analyze the firm's optimization problem within the set of feasible allocations. As a benchmark, online Appendix 1 analyzes cost-minimization in the context of perfect discrimination (the employer can make type-specific take-it-or-leave-it offers to each type of worker).

3.1 Participation and incentive compatibility constraints

If both types are employed, taking the hour constraint into consideration, total cost minimization *does not* involve independently minimizing the cost of hours supplied by the two types since the existence of a contract $P_2 = (c_2, h_2)$ modifies the terms of the incentive compatibility constraint applied to type 1 workers.

The set of constraints includes:

1) Participation constraints (PC):

In the case where there are only two types of workers, $i = (1, 2)$, the participation constraint of each type of worker can be written:

$$u_1 = c_1 - v_1(h_1) \geq 0 \tag{5}$$

$$u_2 = c_2 - v_2(h_2) \geq 0 \tag{6}$$

2) Incentive compatibility (IC) constraints:

Type 1 workers are eligible to contract $P_2 = (c_2, h_2)$; they prefer or accept the contract $P_1 = (c_1, h_1)$ only if:

$$c_1 - v_1(h_1) \geq c_2 - v_1(h_2) \tag{7}$$

Similarly, type 2 workers prefer or accept contract P_2 to contract P_1 only if:

$$c_2 - v_2(h_2) \geq c_1 - v_2(h_1) \quad (8)$$

We can then identify a first property of the contracts.

Proposition 1 *Since the compensation premium increases with working hours h , type 1 workers are offered at least as long hours than type 2 workers, $h_1 \geq h_2$.*

Proof. Adding (7) and (8) :

$$v_2(h_1) - v_1(h_1) \geq v_2(h_2) - v_1(h_2). \quad (9)$$

Because the compensation premium $[v_2(h) - v_1(h)]$ increases with h , the inequality (9) is not compatible with $h_1 < h_2$. ■

This rule is a consequence of the incentive constraints, and it applies not only to optimum values of working hours (cost minimizing values), but also to all feasible values.

The general set of constraints allows us to infer a set of rules:

1) From the IC condition (7), $c_1 - v_1(h_1) \geq c_2 - v_1(h_2)$. The participation constraint for type 2 (6) requires $c_2 \geq v_2(h_2)$. Thus $c_1 - v_1(h_1) \geq v_2(h_2) - v_1(h_2)$, and from our basic assumption (1) $v_2(h_2) - v_1(h_2) > 0$. Consequently:

$$c_1 - v_1(h_1) \geq v_2(h_2) - v_1(h_2) > 0. \quad (10)$$

This arrangement makes the participation constraint for the type 1 workers redundant and indicates the need for a *positive surplus* $[v_2(h_2) - v_1(h_2)]$ for type 1 workers.

2) Type 2 workers who are supposed to be available in any quantity obtain no surplus in the solution; their participation constraint must be saturated:

$$c_2 = v_2(h_2). \quad (11)$$

Limiting c_2 to its minimum value of $v_2(h_2)$ decreases the cost of workers of type 2 directly, and the cost of workers of type 1 indirectly by mitigating the cost of the relevant incentive compatibility constraint.

3) If $c_2 = v_2(h_2)$, the incentive compatibility constraint for type 1 workers (condition 7) becomes :

$$c_1 - v_1(h_1) \geq v_2(h_2) - v_1(h_2). \quad (12)$$

where $v_2(h_2) - v_1(h_2)$ is the surplus obtained by type 1 worker choosing contract P_2 . Therefore, in its saturated form, the type 1 incentive compatibility constraint implies a positive surplus for the type 1 worker:

$$c_1 = v_1(h_1) + [v_2(h_2) - v_1(h_2)]. \quad (13)$$

This situation contrasts with the perfect discrimination case (see online Appendix 1), in which type 1 workers obtain a zero surplus.

4) It must be noted that in our assumptions, the incentive compatibility constraint for type 2 workers is satisfied. Type 2 workers prefer contract $P_2 = (h_2, c_2)$ to contract $P_1 = (h_1, c_1)$ if: $c_2 - v_2(h_2) = 0 \geq c_1 - v_2(h_1)$. Replacing c_1 and c_2 by their expressions in (11) and (13), IC2 becomes $v_2(h_1) - v_1(h_1) \geq v_2(h_2) - v_1(h_2)$, which is true given the rule stated by Proposition 1. This allows us to omit the explicit treatment of condition (8) in the constrained cost minimization problem.

3.2 Cost minimization: first order conditions

In the case where the demand for hours is large enough to justify the hiring of the both types of workers ($H > H_M$) the cost function can be written as:

$$C(n_1, n_2, h_1, h_2, H) = n_1(c_1 + \theta) + n_2(c_2 + \theta). \quad (14)$$

The limited supply of type 1 workers imposes $n_1 \leq \bar{n}_1$. We show in online Appendix 2 that when the two types of workers are employed, this constraint must be saturated. After applying the substitutions introduced in equations (11) and (13), the cost function becomes:

$$C(\bar{n}_1, n_2, h_1, h_2, H) = \bar{n}_1 [v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta] + n_2 [v_2(h_2) + \theta]. \quad (15)$$

The total hours constraint is:

$$n_1 h_1 + n_2 h_2 - H = 0. \quad (16)$$

The firm seeks to minimize the cost $C(\bar{n}_1, n_2, h_1, h_2, H)$ under the hour constraint, by choosing h_1, h_2 and n_2 . To solve this problem, we introduce the Lagrangian:

$$L = \bar{n}_1 [v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta] + n_2 [v_2(h_2) + \theta] - \lambda [\bar{n}_1 h_1 + n_2 h_2 - H]. \quad (17)$$

The first-order conditions applied to interior solutions are as follows:

$$\frac{\partial L}{\partial n_2} = v_2(h_2^*) + \theta - \lambda h_2^* = 0 \quad (18)$$

$$\frac{\partial L}{\partial h_1} = \bar{n}_1 v_1'(h_1^*) - \lambda \bar{n}_1 = 0 \quad (19)$$

$$\frac{\partial L}{\partial h_2} = \bar{n}_1 [v_2'(h_2^*) - v_1'(h_2^*)] + n_2^* v_2'(h_2^*) - \lambda n_2^* = 0 \quad (20)$$

$$\frac{\partial L}{\partial \lambda} = \bar{n}_1 h_1^* + n_2^* h_2^* - H = 0. \quad (21)$$

where $(\lambda^*, n_2^*, h_1^*, h_2^*)$ denotes the solution of the cost minimization problem.

3.3 Properties of the solution

The compensations involved in the two contracts are $c_1^* = v_1(h_1^*) + v_2(h_2^*) - v_1(h_2^*)$ and $c_2^* = v_2(h_2^*)$.

From our assumptions, type 2 workers receive a positive surplus $v_2(h_2^*) - v_1(h_2^*) > 0$.

We can further remark that condition (19) implies: $v_1'(h_1^*) = \lambda$. Thus conditions (18) and (19) imply:

$$\frac{v_2(h_2^*) + \theta}{h_2^*} = v_1'(h_1^*). \quad (22)$$

Equation (22) indicates the equality of the marginal cost of hours obtained from the two possible sources: increasing the number of type 2 workers with constant working time (extensive margin) or increasing the working time of type 1 workers, their number being constant (intensive margin).

a/ Ordering working times

We know from Proposition 1 that $h_1^* \geq h_2^*$. We can show that $h_1^* > h_2^*$ and $h_2^* < \hat{h}_2$.

First, conditions (19) and (20) imply:

$$\bar{n}_1 [v_2'(h_2^*) - v_1'(h_2^*)] + n_2^* [v_2'(h_2^*) - v_1'(h_1^*)] = 0. \quad (23)$$

From condition (2), $v_2'(h_2^*) > v_1'(h_2^*)$, thus condition (23) implies $v_1'(h_1^*) > v_2'(h_2^*)$. Joined, the two conditions imply $v_1'(h_1^*) > v_1'(h_2^*)$. Since $v_1'(h)$ is assumed strictly increasing, $h_1^* > h_2^*$. Consequently, $v_2(h_2^*) - v_1(h_2^*) > 0$ and $c_1^* > c_2^*$.

Second, we prove that $h_2^* < \hat{h}_2$. From (22) and $v_1'(h_1^*) > v_2'(h_2^*)$, we obtain $[v_2(h_2^*) + \theta] / h_2^* > v_2'(h_2^*)$ or $v_2(h_2^*) - h_2^* v_2'(h_2^*) > -\theta$, whereas $v_2(\hat{h}_2) - \hat{h}_2 v_2'(\hat{h}_2) = -\theta$. Since the convex function $\varphi(h) \equiv v_2(h) - h v_2'(h)$ is monotonously decreasing, $h_2^* < \hat{h}_2$.

As a consequence of these shorter hours ($h_2^* < \hat{h}_2$), the principal agrees to pay an extra cost per hour for the type 2 workers (compared to the first best choice), to achieve a reduction in the surplus he/she must grant to type 1 workers.⁵ Intuitively, when the number of target hours H increases, the importance of the type 2 workers also increases, and the principal has lower incentives to accept a higher hourly cost per type 2 worker. The optimal hours h_2^* should increase, and tend toward \hat{h}_2 as $H \rightarrow \infty$.

b/ Ordering wage rates

To each consumption-hours contract is associated an implicit hourly wage. Since no surplus is needed for the participation of type 2, $c_2^* = v_2(h_2^*)$ and the wage rate can be written $w_2 = v_2(h_2^*) / h_2^*$. From (22), $[v_2(h_2^*) + \theta] / h_2^* = v_1'(h_1^*) \Leftrightarrow v_2(h_2^*) / h_2^* = v_1'(h_1^*) - \theta / h_2^*$. Thus $w_2 = v_1'(h_1^*) - \theta / h_2^*$. The wage rate of the type 1 is $w_1 = c_1^* / h_1^*$, where according to his IC constraint (13), $c_1^* = v_1(h_1^*) + \varsigma(h_2^*)$, where $\varsigma(h_2^*) = v_2(h_2^*) - v_1(h_2^*)$ is the compensation premium. The wage rate is $w_1 = [v_1(h_1^*) + \varsigma(h_2^*)] / h_1^*$. Therefore, using the elasticity of compensation with respect to working time $\eta(h_1^*) = \frac{h_1^* v_1'(h_1^*)}{v_1(h_1^*)} > 1$, we can write:

$$w_1 > w_2 \text{ if } \theta \frac{h_1^*}{h_2^*} + \varsigma(h_2^*) > v_1(h_1^*) [\eta(h_1^*) - 1], \quad (24)$$

where h_1^* and h_2^* are endogenous variables. It turns out that the first order conditions (18) to (21) are a priori compatible with *any* wage rate ordering. We will be able to provide additional insights to this important question in Section 5, for a specific compensation structure.

c/ Cost function

The (optimal) cost function, to be obtained by introducing the optimal values $n_2^*(H), h_1^*(H)$ and $h_2^*(H)$ in equation (15) can be written in a compact form as:

$$C^*(H) = C(n_2^*(H), h_1^*(H), h_2^*(H)) \quad (25)$$

⁵ This result is in line with the damaged goods problem introduced by Deneckere and McAfee (1996).

It can be confirmed, after relevant substitutions, that Lagrangian multiplier λ^* reflects the marginal cost of hours, as obtained at the optimum from either type 1 or type 2 workers.

$$\frac{dC^*(H)}{dH} = \lambda^* = v'_1(h_1^*) = \frac{v_2(h_2^*) + \theta}{h_2^*}. \quad (26)$$

4 The case of a linearly increasing compensation premium

Additional insights can be obtained for a simple preference structure that fulfills the condition (2):

$$v_2(h) = v_1(h) + \beta h, \quad (27)$$

where β is a positive constant. Under this assumption, not only is the type 2 worker more demanding than the type 1 (see condition 1), but the compensation premium $[v_2(h) - v_1(h)]$ is *linearly* increasing with h .

We have denoted by \hat{h}_1 and \hat{h}_2 the hours of work that minimize the hourly cost in employing each type of worker. The first order conditions for minimizing the hourly cost are: $v'_1(h) = \frac{v_1(h) + \theta}{h}$ and $v'_2(h) = \frac{v_2(h) + \theta}{h}$. In the linear case, the first order condition for the type 2 become: $v'_1(h) + \beta = \frac{v_1(h) + \beta h + \theta}{h} \Leftrightarrow v'_1(h) = \frac{v_1(h) + \theta}{h}$. Therefore, for this specific function, HCM hours are identical for the two types:

$$\hat{h}_1 = \hat{h}_2 \equiv \hat{h}. \quad (28)$$

4.1 Working time ordering

Proposition 1 shows that if the compensation differences $[v_2(h) - v_1(h)]$ increase with working hours h , then type 1 workers are offered more contract hours than type 2 workers, $h_1 \geq h_2$. We can verify that this general property of the model applies to optimal hours. From assumption (27): $v'_2(h_2^*) = v'_1(h_2^*) + \beta$. Then condition (23) becomes:

$$\bar{n}_1\beta + n_2^*[v'_1(h_2^*) - v'_1(h_1^*) + \beta] = 0 \quad (29)$$

or equivalently:

$$v'_1(h_1^*) - v'_1(h_2^*) = \beta \frac{(\bar{n}_1 + n_2^*)}{n_2^*} \quad (30)$$

Since $\beta > 0$, $v'_1(h_1^*) > v'_1(h_2^*)$. From strict convexity of $v_1(h)$, we have $v'_1(h_1^*) > v'_1(h_2^*) \Rightarrow h_1^* > h_2^*$.

We have shown that $h_2^* < \hat{h}$ in the general case. In the case of the linear increasing difference in compensation we can further show that the optimal hours of type 1 workers exceed their hourly cost-minimizing hours, $\hat{h} < h_1^*$. We obtain that:

Proposition 2 *Optimal contract hours of the type 2 workers are shorter than their hourly-cost minimizing hours, and the optimal contract hours of the type 1 are higher than their hourly-cost minimizing hours.*

$$h_2^* < \hat{h} < h_1^*. \quad (31)$$

Proof. See online Appendix 3. ■

4.2 The consequence of increasing demand for hours

The macroeconomic business cycle leads to fluctuations in the number of hours required by firms. Strong economic expansion is tantamount to a substantial increase in H . From necessary first order conditions applied to (17), it is possible to predict the effect of indefinitely increasing target hours H on optimally contracted working hours.

Proposition 3 *When the total demand for hours H indefinitely increases, for constant \bar{n}_1 , hours h_2^* specified in the optimal contract P_2^* tend to the hourly-cost minimizing value \hat{h} .*

Proof. From equation (16), we find that when the demand for hours increases ($H \rightarrow \infty$), since $n_1 \leq \bar{n}_1$ is fixed and since h_1 and h_2 cannot increase indefinitely, necessarily $n_2 \rightarrow \infty$. From equation (30) $v_1'(h_1^*) - v_1'(h_2^*) = \beta \frac{(\bar{n}_1 + n_2^*)}{n_2^*}$. If $n_2^* \rightarrow \infty$, then $[v_1'(h_1^*) - v_1'(h_2^*)] \rightarrow \beta$. From equation (22), $\frac{v_2(h_2^*) + \theta}{h_2^*} = v_1'(h_1^*)$ and since by assumption, $v_2(h_2^*) = v_1(h_2^*) + \beta h_2^*$, $\frac{v_2(h_2^*) + \theta}{h_2^*} = \frac{v_1(h_2^*) + \beta h_2^* + \theta}{h_2^*} = \frac{v_1(h_2^*) + \theta}{h_2^*} + \beta = v_1'(h_1^*)$. Since $v_1'(h_1^*) \rightarrow v_1'(h_2^*) + \beta$, equation (22) implies: $\frac{v_1(h_2^*) + \theta}{h_2^*} \rightarrow v_1'(h_2^*)$. The limit of this equation is only compatible (uniqueness) with the hourly cost minimizing working time for type 1 workers (and for type 2 workers in our special assumption). Therefore: $H \rightarrow \infty \Rightarrow n_2^* \rightarrow \infty \Rightarrow h_2^* \rightarrow \hat{h}_2 = \hat{h}$. ■

As is intuitively predictable, when the proportion of type 2 workers indefinitely increases, the external effect on their contract by type 1 worker preferences dwindles, and type 2 workers' working time tends towards its HCM value.

Proposition 4 *When the total demand for hours H indefinitely increases, hours h_1^* in optimal contract P_1 always exceed the hourly-cost minimizing hours \hat{h} .*

Proof. With respect to the evolution of contract P_1^* we note that from equation (30) $[v_1'(h_1^*) - v_1'(h_2^*)] \rightarrow \beta$ and simultaneously, $h_2^* \rightarrow \hat{h} = \hat{h}_1$ implying: $v_1'(h_1^*) \rightarrow v_1'(\hat{h}) + \beta$ and therefore $h_1^* > \hat{h}$. ■

The working time for type 1 workers is kept above its HCM value under the influence of β , the parameter that captures the higher sensitivity of type 2 workers with respect to hours (the compensation premium).

5 The case of quadratic compensation functions

The specific compensations functions used above allowed us to better characterize the optimal contract hours. Introducing an even more precise structure for the compensation functions enables us to fully characterize the solution and numerically solve the model and reveal a possible paradoxical ordering of hourly wage rates. We also resort to a numerical simulation to provide additional intuition about the properties of the solution.

Quadratic function $v(h) = h^2$ has the required property of strict convexity. Our compensating consumption functions then become:

$$v_1(h) = h^2 \text{ and } v_2(h) = h^2 + \beta h. \quad (32)$$

We have shown that for a linear increasing compensation premium, the hourly cost-minimizing hours \hat{h}_1 and \hat{h}_2 are identical: $\hat{h}_1 = \hat{h}_2 = \hat{h}$. With a quadratic compensation function, $\hat{h} = \sqrt{\theta}$.

5.1 The solution to the cost minimization problem

With this assumption, the equations (23) and (22) become:

$$(h_2^*)^2 + \beta h_2^* + \theta = 2h_1^* h_2^* \quad (33)$$

$$\beta (\bar{n}_1 + n_2^*) + 2n_2^* (h_2^* - h_1^*) = 0 \quad (34)$$

To which we add the hours constraint:

$$\bar{n}_1 h_1^* + n_2^* h_2^* = H \quad (35)$$

Equations (33), (34) and (35) form a nonlinear system with three endogenous variables.

The linearity of the subsystem (35 and 34) for a given value of n_2^* enables the expression of h_1 and h_2 as functions of the endogenous employment of type 2 workers n_2^* , yielding a feature of the

solutions in comparison to the average working time $\bar{h} = H/(\bar{n}_1 + n_2^*)$:

$$h_1^* = \bar{h} + \frac{\beta}{2} \quad (36)$$

$$h_2^* = \bar{h} - \frac{\beta \bar{n}_1}{2 n_2^*} \quad (37)$$

We emphasize a first property of the solution. Equation (33) has distinct real roots only if its determinant $(\beta - 2h_1^*)^2 - 4\theta > 0$, implying $\beta - 2h_1^* > 2\sqrt{\theta}$ or $\beta - 2h_1^* < -2\sqrt{\theta}$. Since $\bar{h} > 0$, the first case is not compatible with (36). The second case is consistent with (36) for $\bar{h} > \sqrt{\theta}$. Since hourly-cost minimizing hours for both types are defined by $\hat{h} = \sqrt{\theta}$, first order conditions imply $\bar{h} > \hat{h}$.

Introducing optimal hours as defined in equations (36) and (37) in equation (33), it turns out that:

Proposition 5 *The optimal number n_2^* of P_2 contracts offered by the employer (n_2^*) is the superior root of:*

$$\frac{H^2}{(\bar{n}_1 + n_2^*)^2} = \theta + \left(\frac{\beta \bar{n}_1}{2 n_2^*} \right)^2. \quad (38)$$

Proof. See online Appendix 4 for the full proof. In brief, we show that there is a value $H = H^o$ for which equation (38) has a single root, $n_2^o = \bar{n}_1 \sqrt[3]{\frac{\beta^2}{4\theta}}$. We also show that for $H < H^o$, the equation (38) has no real root, and for $H > H^o$ it has two positive roots, n_2^l and n_2^h , with $n_2^l < n_2^o < n_2^h$. Furthermore, online Appendix 5 reveals the second order necessary condition for cost minimization:

$$\left(\frac{n_2^*}{\bar{n}_1} \right)^3 \geq \frac{\beta^2}{4\theta} \Leftrightarrow n_2^* \geq n_2^o \quad (39)$$

We conclude that only the superior root n_2^h satisfies the *second order necessary conditions* for cost minimization. ■

Corollary 6 *For $n_2 < n_2^o$, the cost minimization problem has no solution. Any mixed employment regime involves a minimum number of type 2 workers determined by parameters in inequality (39), as associated to a minimum amount of hours H^o .*

The threshold hours H^o is obtained from equation (38) evaluated for n_2^o .

Numerical simulations and graphic representations of the relevant functions in Appendix 4 confirm that the superior root corresponds to the minimum of the cost function (i.e., our solution n_2^*).

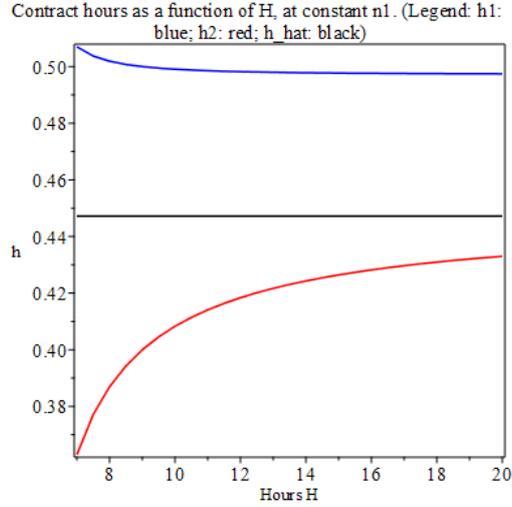


Figure 1: Contract hours as a function of H

For n_2^* as defined by equation (38), equations (36) and (37) allow us to determine h_1^* and h_2^* , thus fully characterizing the solution to the cost-minimization problem.

5.2 Comparative statics

It is interesting to study how h_1^* , h_2^* and n_2^* evolve when the target hours H increases. We can show that:

Proposition 7 *For interior solutions, n_2^* i.e., the optimal number of P_2 contracts (demand for type 2 workers), is increasing with H .*

Proof. See online Appendix 6. ■

This comparative static rule could appear intuitively trivial; however, it is not, since for an increased hours constraint, the employer also controls the two working times h_1 and h_2 .

A numerical simulation allows us to study how the optimal working hours vary when H increases. The parameters used are $n_1 = 10$, $\beta = 0.10$ and $\theta = 0.20$. We apply $H \geq 7$, as we numerically verify (in 5.4) that for $H < 7$ the employer prefers to hire only the type 1 workers.

Figure 1 displays h_2^* as the red, lower curve and h_1^* as the blue, upper curve. The horizontal line corresponds to the hourly cost minimizing hours of type 2 workers, $\hat{h} = \sqrt{\theta} = 0.447$. In line with Proposition 3, when $H \rightarrow \infty$, $h_2^* \rightarrow \hat{h}$. In line with Proposition 4, when $H \rightarrow \infty$, $v_1'(h_1^*) \rightarrow v_1'(\hat{h}) + \beta$ or $h_1^* \rightarrow \hat{h} + 0.5\beta = 0.497$.

5.3 Wage rate inequality

We analyze conditions to obtain $w_1 > w_2$, i.e., a paradoxical situation in which the less demanding type of worker obtains a higher wage rate than the more demanding type of worker.

The wage rate being $w_1 = \frac{v_1(h_1) + v_2(h_2) - v_1(h_2)}{h_1}$ and $w_2 \equiv \frac{v_2(h_2)}{h_2}$, the inequality can be written as:

$$\frac{v_1(h_1) + v_2(h_2) - v_1(h_2)}{h_1} > \frac{v_2(h_2)}{h_2} \iff \frac{v_1(h_1) - v_1(h_2)}{h_1 - h_2} > \frac{v_2(h_2)}{h_2}. \quad (40)$$

In the quadratic case $v_1(h_1) = h_1^2$ and $v_2(h_2) = h_2^2 + \beta h_2$ so:

$$w_1 > w_2 \iff \frac{h_1^2 - h_2^2}{h_1 - h_2} > \frac{h_2^2 + \beta h_2}{h_2} \iff h_1 + h_2 > h_2 + \beta \iff h_1 > \beta. \quad (41)$$

From (36), $h_1 = h_1^* = \bar{h} + \frac{\beta}{2}$, therefore

$$w_1 > w_2 \iff \bar{h} > \frac{\beta}{2}, \quad (42)$$

where $\bar{h} \equiv \frac{H}{\bar{n}_1 + n_2}$ is a dependent variable.

We can reveal two sufficient conditions for this specific wage ordering to be observed.

a/ From equation (38) $\bar{h}^2 = \theta + \left(\frac{\beta \bar{n}_1}{2n_2}\right)^2$ implying $\bar{h}^2 > \theta \Rightarrow \bar{h} > \sqrt{\theta}$. It turns out that $\sqrt{\theta} > \frac{\beta}{2}$ is a *sufficient condition in terms of parameters* for $w_1 > w_2$. Under our assumptions, chances to observe higher wages for the less demanding workers would be higher in sectors that involve a large fixed cost per worker.

b/ Remembering the second order condition (39), we have $\left(\frac{n_2^*}{\bar{n}_1}\right)^3 > \frac{\beta^2}{4\theta}$ or $\sqrt{\theta} > \frac{\beta}{2} \left(\frac{\bar{n}_1}{n_2}\right)^{3/2}$. Once again, we recall that equation (38) implies $\bar{h} > \sqrt{\theta}$ and $\sqrt{\theta} > \frac{\beta}{2} \left(\frac{\bar{n}_1}{n_2}\right)^{3/2}$. Thus, another *sufficient condition in terms of dependent variables* is $\frac{\beta}{2} \left(\frac{\bar{n}_1}{n_2}\right)^{3/2} > \frac{\beta}{2} \iff n_2 < \bar{n}_1$. The wage rate ordering $w_1 > w_2$ should be observed at least when $n_2^* < \bar{n}_1$.

Figure 2 shows the evolution in hourly wages as the demand for hours H increases ($n_1 = 10, \beta = 0.10$ and $\theta = 0.20$; the condition $\sqrt{\theta} > 0.5\beta$ holds). The upper, blue curve indicates the wage rate of type 1 workers, and the lower, red curve indicates the wage rate of the type 2 workers.

The simulation confirms that the wage rate of the less demanding workers (who also work longer hours) is higher than the wage rate the more demanding workers. The wage gap favorable

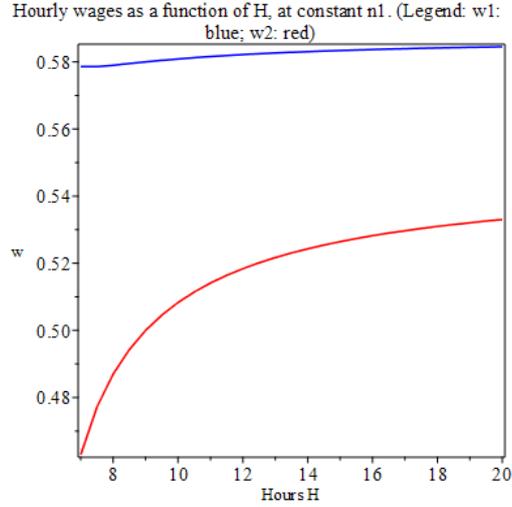


Figure 2: Wage rates by worker type, depending on H

to type 1 workers is basically explained by the surplus that these workers obtain in the mixed employment regime when the employer cannot discriminate types.

This result is contrasting with the perfect discrimination situation as depicted in online Appendix 1. In that case, with quadratic compensation functions, the optimal contracts involve longer working hours and a *lower wage rate* for the less demanding workers compared to the more demanding workers. Furthermore, these wage rates are independent of the total number of hours H , while they increase with H in the nondiscriminating case.

As mentioned in the introduction, several scholars (Goldin, 2014; Cortés and Pan, 2019) provide evidence that in some high-skilled occupations long working hours are associated with higher hourly wages. The authors attribute this difference to the higher productivity of employees working longer hours, associated to better serving of clients who require "temporal flexibility" and a better within-firm circulation of information. In our model, the use of a specific compensation structure can generate a *higher wage rate* to the *less demanding type 1* workers (who work longer hours) while not resorting to any productivity argument.

5.4 Cost analysis

Finally, we would like to ensure that our simulations adequately match the case in which the employer prefers to use both type of workers rather than to use only the less demanding type 1.

More precisely, we must verify that the hours H used in the simulation are great enough to justify the employment of both types of workers.

In the quadratic compensation case, $v_1(h) = h^2$, the hourly cost minimizing working hours of these least demanding workers is $\hat{h}_1 = \sqrt{\theta}$. As long as $H < H_1 = \bar{n}_1 \sqrt{\theta}$ (in our simulation, $H_1 = 10\sqrt{0.20} \approx 4.5$), the firm should offer only the contract $(v_1(\hat{h}_1), \hat{h}_1)$ and employ only type 1 workers. For $H > H_1$, the cost of labor when employing only type 1 workers becomes convex, $C_1(H) = \bar{n}_1 \left[\left(\frac{H}{\bar{n}_1} \right)^2 + \theta \right]$.⁶ For a large H the firm *might* want to hire both type of workers, provided that the cost of doing so is lower than using only type 1.

The cost of labor when using both type of workers, denoted by $C^*(H)$, has been defined in (25). With quadratic compensation functions, the cost function becomes:

$$C^*(H) = (\bar{n}_1 + n_2^*)\theta + \bar{n}_1 \left[(h_1^*)^2 + \beta h_2^* \right] + n_2^* \left[(h_2^*)^2 + \beta h_2^* \right] \quad (43)$$

where n_2^* , h_1^* and h_2^* are optimal values as resulting from the cost minimization problem (they all depend on H).

To determine the threshold total hours for which the mixed employment regime dominates the type 1 only employment regime, we compare the cost of using only type 1 workers, $C_1(H)$ with the cost of labor under the mixed regime $C^*(H)$.

Our numerical simulations show that for $H \geq H_M = 7$ the mixed employment regime is indeed less expensive than employing only type 1 workers, as it can be seen in Figure 3, where $C_1(H)$ is displayed as the upper black, curve and $C^*(H)$ as the lower, green curve. Below $H = 7$, the cost of hiring only type 1 workers is lower than the cost of the mixed employment.

We argued in corollary 6 that the cost minimization problem has a interior solution (for the mixed employment regime) only if $H \geq H^o$. We can verify that $H^o = 6.12$ thus the solution exists for $H \geq 7$.

The comparison of costs functions across employment regimes reveals the emergence of a *discontinuity* in the demand for type 2 workers. Indeed, for $H < 7$, $n_2^* = 0$; for $H = 7$, $n_2^* = 5.32$, and for $H > 7$, $n_2^* > 5.32$. This feature of the model can be extremely problematic in periods of

⁶ At some point, it might even exceed the cost of employing only type 2 workers.

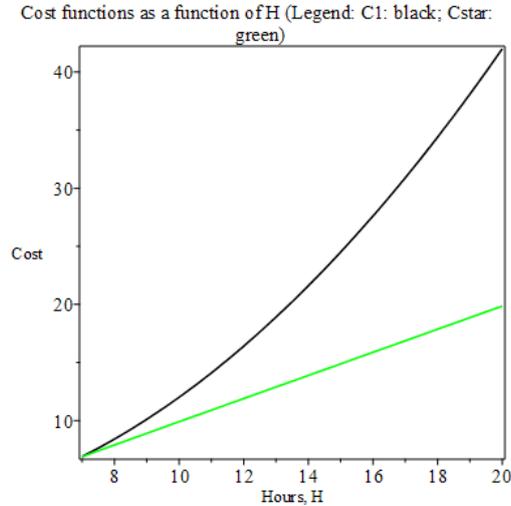


Figure 3: Cost functions

economic crisis, as smooth downward changes in the total demand for hours can translate into massive cuts to the employment of type 2 workers.

6 Conclusion

In being confronted with a wealth of data exhibiting frequent discrepancies in labour contracts, especially including compensation differences for apparently equivalent workers, many researchers have provided relevant explanations involving discriminating policies. This paper does not contradict this literature, but suggests the existence of an alternative explanation based on standard contract theory.

In the proposed model, a firm seeks a predetermined volume of homogenous working hours, to be provided by workers who value leisure time to greater and lesser degree. Our analysis focuses on a nontrivial case in which less demanding workers are in scarce supply. Discrimination is forbidden, i.e., the employer cannot offer a specific contract to each type of worker. In this context, the contract offered to the more demanding workers performs as a fixed cost of hiring the less demanding workers, a mechanism that ensures efficient self-selection of the workers. As a consequence, the scarce less demanding workers can actually be offered higher wage rates than their more demanding peers.

These results reveal wage differentials originating "from nothing", rooted in an invisible scarcity

constraint impossible to trace by econometrics from individual data. If one agrees that women have better work alternatives outside the labor market (Cain, 1986; OECD, 2018), and demand higher compensation than men for any given working time, in our model the (hourly) wage gender gap in favour of male workers could be grounded in these differences in preferences and the scarcity of male workers, and not in differences in productivity, in negative perceptions or in women penalizing stereotypes. This interpretation of course does not address the important question of why the burden of child care falls in a significant way on women.

We have also shown how contract hours depend on the structure of preferences; for a time increasing compensation premium, the less demanding workers will do longer hours than the more demanding workers.

Finally, the model allowed us to analyze how working time, hourly wages and employment respond in a somewhat unconventional way to changes in the total demand for hours. In particular, we show that the demand for the most demanding, abundant workers is exposed to discontinuities. From a macroeconomic perspective, the discontinuity in the demand for the most demanding workers can explain why small fluctuations in global demand are sometimes associated with large fluctuations in some types of employment.

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1 APPENDIX

1.1 The case of perfect discrimination

We study a cost-minimization problem similar to the problem addressed in the main text. There are two types of workers characterized by non-crossing compensation functions, with $v_2(h) > v_1(h)$, and an increasing compensation premium $v_2(h) - v_1(h)$. Type 1 workers are in scarce supply, let \bar{n}_1 be their maximum number. The demand of hours H is large enough to justify the employment of the type 2. *However, in this case, the employer, who knows the worker's type, can make take-it-or-leave-it job offers to each of them.* The type-specific contract includes hours of work and the compensation. In the perfect discrimination condition the firm can offer zero-surplus contracts to both types ($c_1 = v_1(h_1)$ and $c_2 = v_2(h_2)$)

Let us denote the solution to this different cost minimization problem by h_1^a , h_2^a and n_2^a .

The total cost is:

$$C = \bar{n}_1 [v_1(h_1) + \theta] + n_2 [v_2(h_2) + \theta] \quad (44)$$

and the hour constraint:

$$\bar{n}_1 h_1 + n_2 h_2 - H \quad (45)$$

The relevant Lagrangian for the cost minimization under the hour constraint is:

$$L(h_1, h_2, n_2, \lambda) = \bar{n}_1 [v_1(h_1) + \theta] + n_2 [v_2(h_2) + \theta] - \lambda [\bar{n}_1 h_1 + n_2 h_2 - H]. \quad (46)$$

First order conditions are:

$$L_{h_1} = \bar{n}_1 [v_1'(h_1^a) - \lambda^a] = 0 \quad (47)$$

$$L_{h_2} = n_2 [v_2'(h_2^a) - \lambda^a] = 0 \quad (48)$$

$$L_{n_2} = v_2(h_2^a) + \theta - \lambda^a h_2^a = 0 \quad (49)$$

$$L_\lambda = (\bar{n}_1 h_1^a + n_2 h_2^a - H) = 0. \quad (50)$$

From the third condition we have $\lambda^a = \frac{v_2(h_2^a) + \theta}{h_2^a}$. Combined with the second one, this involves:

$$v_2'(h_2^a) = \frac{v_2(h_2^a) + \theta}{h_2^a}. \quad (51)$$

This condition is identical to FOC for hourly cost minimization for type 2. The first-best optimal hours of the type 2 correspond to the hourly-cost minimizing hours, $h_2^a = \hat{h}_2$, a result which is in line with intuitive reasoning. Since these workers are available in any amount, and the contract offered to type 1 is independent of the contract offered to type 2, the firm offers to them the contract that minimizes their hourly cost.

From equations (47) and (48) we obtain:

$$v_1'(h_1^a) = v_2'(\hat{h}_2) = \frac{v_2(\hat{h}_2) + \theta}{\hat{h}_2} \quad (52)$$

At the optimum, the marginal cost from increasing the hours provided by the type 1 (intensive margin) must be identical to the marginal cost of hiring more of the type two workers (extensive margin) (at the lowest hourly cost for the later).

The first-best contract hour for the type 1, denoted by h_1^a are larger than the cost minimizing hours \hat{h}_1 . Indeed, for any convex functions $v_2(h), v_1(h)$ and $v_2(h) > v_1(h)$, a simple graphical analysis can show that:

$$v_2'(\hat{h}_2) = \frac{v_2(\hat{h}_2) + \theta}{\hat{h}_2} > \frac{v_1(\hat{h}_1) + \theta}{\hat{h}_1} = v_1'(\hat{h}_1) \quad (53)$$

But $v_1'(h_1^a) = v_2'(\hat{h}_2)$ thus $v_1'(h_1^a) > v_1'(\hat{h}_1) \Leftrightarrow h_1^a > \hat{h}_1$.

Finally, the hours constraint $n_2^a h_2^a + \bar{n}_1 h_1^a = H$ allows us to determine n_2^a as a function of H .

An example

To bring more intuition to this analysis of perfect discrimination, we study the specific compensation functions $v_1(h) = h^2$ and $v_2(h) = h^2 + \beta h$. The associated hourly cost minimizing hours are $\hat{h}_1 = \hat{h}_2 = \sqrt{\theta}$.

In this case, expression (51) becomes $\frac{(h_2^a)^2 + \beta h_2^a + \theta}{h_2^a} = 2h_2^a + \beta$ leading to first-best contract hours for the type 2 workers $h_2^a = \hat{h}_2 = \sqrt{\theta}$.

To determine contract hours of the type 1 workers, we set $v_1'(h_1^a) = v_2'(h_2^a) \Rightarrow 2h_1^a = 2h_2^a + \beta = 2\sqrt{\theta} + \beta$ leading to first-best hours $h_1^a = h_2^a + \frac{\beta}{2} = \sqrt{\theta} + \frac{\beta}{2}$.

We can determine the optimal wage rates:

$$w_1^a = \frac{v_1(h_1^a)}{h_1^a} = \frac{(h_1^a)^2}{h_1^a} = h_1^* = \sqrt{\theta} + \frac{\beta}{2}. \quad (54)$$

$$w_2^a = \frac{v_2(\hat{h}_2)}{\hat{h}_2} = \frac{(\hat{h}_2)^2 + \beta\hat{h}_2}{\hat{h}_2} = \sqrt{\theta} + \beta. \quad (55)$$

We verify that: $w_2^a > w_1^a$, the wage rate of the more demanding type is higher than the wage rate of the less demanding type.

1.2 Binding limitation of type 1 workers

In the general case (as analyzed in the main text), considering the problem:

$$\min_{n_1, n_2, h_1, h_2} \{n_1 [v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta] + n_2 [v_2(h_2) + \theta]\} \quad (56)$$

with $[n_1 h_1 + n_2 h_2 - H]$ and $n_1 \leq \bar{n}_1$. The corresponding Lagrangian is:

$$L(n_1, n_2, h_1, h_2, \lambda) = n_1 [v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta] + n_2 [v_2(h_2) + \theta] - \lambda [n_1 h_1 + n_2 h_2 - H]. \quad (57)$$

If $0 < n_1^* < \bar{n}_1$, first order necessary conditions imply:

$$\frac{\partial L}{\partial n_1} = [v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta] - \lambda h_1 = 0 \quad (58)$$

$$\frac{\partial L}{\partial n_2} = v_2(h_2) + \theta - \lambda h_2 = 0 \quad (59)$$

$$\frac{\partial L}{\partial h_1} = n_1 v_1'(h_1) - \lambda n_1 = 0 \quad (60)$$

$$\frac{\partial L}{\partial h_2} = n_1 [v_2'(h_2) - v_1'(h_2)] + n_2 v_2'(h_2) - \lambda n_2 \quad (61)$$

The first three FOCs allow us to write:

$$\frac{v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta}{h_1} = \frac{v_2(h_2) + \theta}{h_2} = \lambda = v_1'(h_1). \quad (62)$$

This equality indicates that if the availability constraint is not binding, the cost of hours obtained from each type should coincide.

But (62) implies:

$$\frac{v_1(h_1) - v_1(h_2)}{h_1 - h_2} = \lambda = v_1'(h_1). \quad (63)$$

(we substitute $v_2(h_2) + \theta = h_2 v_1'(h_1)$ in the first term).

Convexity of $v_1(h)$ implies $v_1(h_2) - v_1(h_1) > (h_2 - h_1)v_1'(h_1) \Rightarrow v_1(h_1) - v_1(h_2) < (h_1 - h_2)v_1'(h_1)$
or $\frac{v_1(h_1) - v_1(h_2)}{h_1 - h_2} < v_1'(h_1)$, contradicting (63).

1.3 Proof of the inequality $h_2^* < \hat{h} < h_1^*$

We have defined by \hat{h}_i the working time that minimizes the hourly cost of each type i regardless of any other constraint, $\hat{h}_1 = \arg \min \left[\frac{v_1(h) + \theta}{h} \right]$ and $\hat{h}_2 = \arg \min \left[\frac{v_2(h) + \theta}{h} \right]$. The first order condition for minimizing the hourly cost are: $v_1'(h) = \frac{v_1(h) + \theta}{h}$ and $v_2'(h) = \frac{v_2(h) + \theta}{h}$ leading to hourly-cost minimizing \hat{h}_1 and \hat{h}_2 . In the linear case $v_2 = v_1(h) + \beta h$, the first order conditions become: $v_1'(h) + \beta = \frac{v_1(h) + \beta h + \theta}{h} \Leftrightarrow v_1'(h) = \frac{v_1(h) + \theta}{h}$. Therefore, in the case of the linear increasing differences, the hourly-cost minimizing hours are identical for the two types:

$$\hat{h}_1 = \hat{h}_2 \equiv \hat{h}. \quad (64)$$

Let us recall here the main text equation (22):

$$\frac{v_2(h_2^*) + \theta}{h_2^*} = v_1'(h_1^*) \quad (65)$$

and main text equation (30):

$$v_1'(h_1^*) - v_1'(h_2^*) = \beta \frac{(\bar{n}_1 + n_2^*)}{n_2^*} \quad (66)$$

a/ To show that $h_1^* > \hat{h}$ we start from the definition of \hat{h}_2 and use equality $\hat{h}_1 = \hat{h}_2 = \hat{h}$. Uniqueness of \hat{h}_2 implies:

$$\frac{v_2(h_2^*) + \theta}{h_2^*} \geq \frac{v_2(\hat{h}_2) + \theta}{\hat{h}_2} = v_2'(\hat{h}_2). \quad (67)$$

Given that (equation 65) $\frac{v_2(h_2^*) + \theta}{h_2^*} = v_1'(h_1^*)$, inequality (67) implies $v_1'(h_1^*) \geq v_2'(\hat{h}_2)$. Since $\hat{h}_1 = \hat{h}_2 = \hat{h}$ and $v_2'(h) = v_1'(h) + \beta$, inequality (67) implies $v_1'(h_1^*) > v_1'(\hat{h})$ and (from strict convexity), $h_1^* > \hat{h}$.

b/ It also can be shown that $h_2^* < \hat{h}$. From equation (66), $v_1'(h_2^*) = v_1'(h_1^*) - \frac{(\bar{n}_1 + n_2^*)}{n_2^*} \beta$. By assumption, $v_2(h) = v_1(h) + \beta h$, and from equation (65): $\frac{v_1(h_2^*) + \theta}{h_2^*} + \beta = v_1'(h_1^*)$. Then equation (66) implies:

$$v_1'(h_2^*) = \frac{v_1(h_2^*) + \theta}{h_2^*} + \beta - \frac{(\bar{n}_1 + n_2^*)}{n_2^*} \beta. \quad (68)$$

Therefore, $v_1'(h_2^*) < \frac{v_1(h_2^*) + \theta}{h_2^*}$. Considering simultaneously $v_1'(h_2^*) < \frac{v_1(h_2^*) + \theta}{h_2^*}$ and $v_1'(\hat{h}) = \frac{v_1(\hat{h}) + \theta}{\hat{h}}$, it implies:

$$v_1(h_2^*) - h_2^* v_1'(h_2^*) > -\theta \text{ and } v_1(\hat{h}) - \hat{h} v_1'(\hat{h}) = -\theta. \quad (69)$$

It can be shown that the function $\varphi(h) \equiv v(h) - hv'(h)$ is monotonously decreasing, thus equation (69) implies: $h_2^* < \hat{h}$.

1.4 The optimal number of type 2 workers

The computed values of working times for a given (endogenous) number of type 2 workers are:

$$h_1^* = \bar{h} + \frac{\beta}{2} \quad (70)$$

$$h_2^* = \bar{h} - \frac{\beta \bar{n}_1}{2 n_2} \quad (71)$$

where $\bar{h} = \frac{H}{(\bar{n}_1 + n_2)}$. Introducing these expressions in the main text equation $h_2^2 + \beta h_2 + \theta = 2h_2 h_1$ we obtain:

$$\underbrace{\frac{H^2}{(\bar{n}_1 + n_2)^2}}_{L(n_2)} = \theta + \underbrace{\left(\frac{\beta \bar{n}_1}{2 n_2}\right)^2}_{R(n_2)} \quad (72)$$

The demand for type 2 workers, n_2^* , is the implicit solution to the former equation.

Figure 4 represents the functions $L(n_2)$ and $R(n_2)$ for $H = 7$, which is the downward bound for existence of the mixed employment regime. The smaller H , the lower the curve $L(n_2)$ is. The other parameters are similar to those used in the main text ($\beta = 0.10$, $\theta = 0.20$, $\bar{n}_1 = 10$). As we can see, in this case, equation (72) has two positive roots, $n_2^l = 1.13$ and $n_2^h = 5.32$.

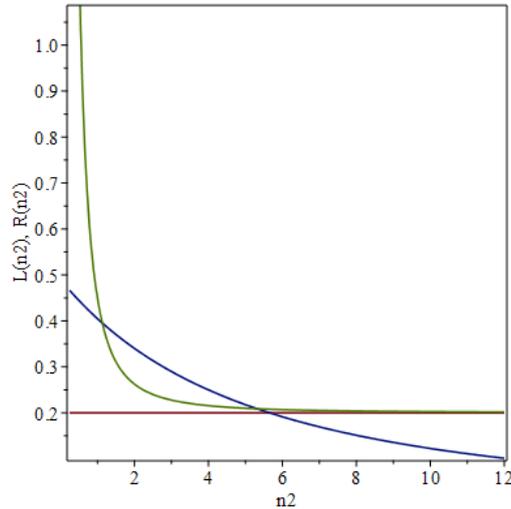


Figure 4: The two positive roots of equation 72

The model has no solution if H is too low since the two curves do not cross. To determine this critical threshold, denoted by H^o , we acknowledge, from (graphic) analysis of $L(n_2)$ and $R(n_2)$,

that the positive real root of (72) is unique if $L(n_2) = R(n_2)$ and $L'(n_2) = R'(n_2)$. This unique root is obtained for H^o . For $H > H^o$ the system has two roots (the superior being our solution), for $H < H^o$ there is no solution.

Derivatives with respect n_2 are:

$$L'(n_2) = -2 \frac{H^2}{(\bar{n}_1 + n_2)^3} \quad (73)$$

$$R'(n_2) = -2 \left(\frac{\beta \bar{n}_1}{2} \right)^2 \frac{1}{(n_2)^3} \quad (74)$$

leading to:

$$L'(n_2) = R'(n_2) \Leftrightarrow \frac{H^2}{(\bar{n}_1 + n_2)^2} = \left(\frac{\beta \bar{n}_1}{2} \right)^2 \frac{(\bar{n}_1 + n_2)}{(n_2)^3} \quad (75)$$

Substituting the first term in equation (72), we obtain:

$$\left(\frac{n_2}{\bar{n}_1} \right)^3 = \frac{\beta^2}{4\theta}. \quad (76)$$

The possible unique positive root, denoted by n_2^0 , is defined by (76) $\left(\frac{n_2^0}{\bar{n}_1} \right)^3 = \frac{\beta^2}{4\theta}$, or $n_2^0 = \bar{n}_1 \sqrt[3]{\frac{\beta^2}{4\theta}} > 0$. We conclude that the mixed employment regime always requires a quantum of type 2 workers. H^0 is hour demand for which $n_2^* = n_2^0$.

For $H > H^0$, equation (72) has two real positive roots, n_2^l and n_2^h . However, the lowest root is smaller than n_2^0 implying $\left(\frac{n_2^l}{\bar{n}_1} \right)^3 < \frac{\beta^2}{4\theta}$. This is contradicting the second order condition for cost minimization (85) as shown in Appendix 1.5.

As a last check, we acknowledge that the cost of producing a predetermined amount of hours can be written as a function of n_2 only, all other variables being set at their optimal level (depending on n_2):

$$C^*(n_2) = (\bar{n}_1 + n_2)\theta + \bar{n}_1 [(h_1^*)^2 + \beta h_2^*] + n_2 [(h_2^*)^2 + \beta h_2^*] \quad (77)$$

where $h_1^* = h_1^*(n_2)$ and $h_2^* = h_2^*(n_2)$ are provided in equations (36) and (37).

Figure 5 displays the plot of the cost as a function of n_2 for $n_1 = 10, \beta = 0.10, \theta = 0.20$ and $H = 7$. The graph corroborates that the larger root ($n_2^h = 5.32$) is the solution to the cost minimization problem.

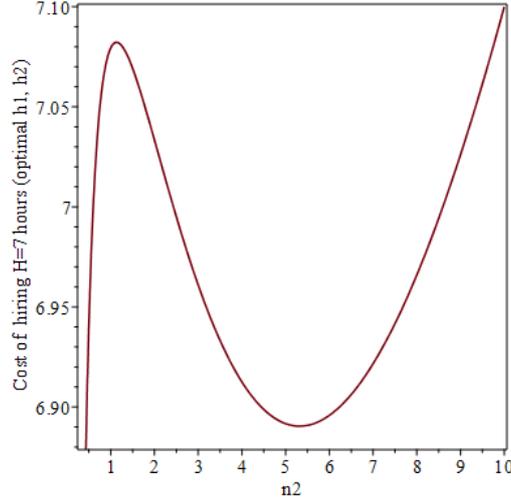


Figure 5: Cost as a function of n_2 , for optimal hours and wage rates

1.5 Second order conditions for cost minimization

We use the hours constraint to reduce the cost-minimization problem to a form of free variables minimization. Since for $n_1 = \bar{n}_1$, n_2 is explicitly determined by the hours constraint and the choice of h_1 and h_2 , and the objective function to be minimized is:

$$m(h_1, h_2) = \bar{n}_1 [v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta] + \frac{H - \bar{n}_1 h_1}{h_2} [v_2(h_2) + \theta]. \quad (78)$$

First order derivatives are:

$$m_1(h_1, h_2) = \bar{n}_1 v_1'(h_1) - \frac{\bar{n}_1}{h_2} [v_2(h_2) + \theta] \quad (79)$$

$$m_2(h_1, h_2) = \bar{n}_1 [v_2'(h_2) - v_1'(h_2)] - \frac{H - \bar{n}_1 h_1}{h_2^2} [v_2(h_2) + \theta] + \frac{H - \bar{n}_1 h_1}{h_2} v_2'(h_2). \quad (80)$$

After substitutions, since $v_1(h) = h^2$, $v_2(h) = h^2 + \beta h$ and $\frac{H - \bar{n}_1 h_1}{h_2} = n_2$, first order conditions are:

$$m_1(h_1, h_2) = 2\bar{n}_1 h_1 - \bar{n}_1 h_2 - \beta \bar{n}_1 - \bar{n}_1 \frac{\theta}{h_2} = 0 \quad (81)$$

$$m_2(h_1, h_2) = \bar{n}_1 \beta + n_2 \left(1 - \frac{\theta}{h_2}\right) = 0. \quad (82)$$

where $\frac{H - \bar{n}_1 h_1}{h_2} = n_2$. From (81) and (82), the Hessian matrix is:

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} 2\bar{n}_1 & \bar{n}_1 \left(\frac{\theta}{h_2^2} - 1\right) \\ \bar{n}_1 \left(\frac{\theta}{h_2^2} - 1\right) & \frac{2n_2}{h_2^2} \theta \end{pmatrix}. \quad (83)$$

Second order necessary conditions for minimization are: $m_{11} > 0$ and $m_{22} > 0$ (always fulfilled),

and $\det \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} > 0$ which is fulfilled iff:

$$\frac{4\bar{n}_1 n_2 \theta}{h_2^2} > \bar{n}_1^2 \left(\frac{\theta}{h_2} - 1 \right)^2 \Leftrightarrow 4\theta n_2 > \bar{n}_1 \left(\frac{\theta}{h_2} - h_2 \right)^2. \quad (84)$$

From (81) $\frac{\theta}{h_2} = 2h_1 - h_2 - \beta$ and $\frac{\theta}{h_2} - h_2 = 2(h_1 - h_2) - \beta$ and from (81) and (82) $2(h_1 - h_2) - \beta =$

$\frac{\bar{n}_1}{n_2} \beta$. Thus $\det \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} > 0$ iff $4\theta n_2 > \bar{n}_1 \left(\frac{\bar{n}_1}{n_2} \beta \right)^2$.

The solution to the system of first order conditions corresponds to a minimum of the cost function if the solution verifies:

$$\left(\frac{n_2^*}{\bar{n}_1} \right)^3 > \frac{\beta^2}{4\theta}. \quad (85)$$

This condition amounts to a minimum relative participation of type 2, $n_2^* = \bar{n}_1 \sqrt[3]{\frac{\beta^2}{4\theta}}$ and a minimum amount of total hours, H^o .

In our simulation, for $\beta = 0.10$, $\theta = 0.20$, $\bar{n}_1 = 10$, we obtain $n_2^o = 2.32$ and $H^o = 6.11$ (we also verify that $H^o < H_M = 7$).

1.6 The relationship between the optimal number of type 2 workers and H

We prove in this Appendix that $\frac{dn_2^*}{dH} > 0$.

From first order conditions, we obtain equation (38) in the main text: $\frac{H^2}{(\bar{n}_1 + n_2^*)^2} = \theta + \left(\frac{\beta \bar{n}_1}{2 n_2^*} \right)^2$.

In the neighborhood of an interior solution, n_2^* is a continuous and differentiable function of H , noted $g(H)$ and the equation can be written:

$$H^2 [\bar{n}_1 + g(H)]^{-2} = \theta + \left(\frac{\beta \bar{n}_1}{2} \right)^2 g^{-2}(H). \quad (86)$$

After derivating both members with respect to H :

$$g'(H) = \frac{H [\bar{n}_1 + g(H)]}{H^2 [\bar{n}_1 + g(H)]^{-3} - \left(\frac{\beta \bar{n}_1}{2} \right)^2 g^{-3}(H)}. \quad (87)$$

The condition for $\frac{dn_2^*}{dH} = g'(H) > 0$ is:

$$H^2 [\bar{n}_1 + g(H)]^{-3} > \left(\frac{\beta \bar{n}_1}{2} \right)^2 g^{-3}(H) \quad (88)$$

or equivalently:

$$H^2 [\bar{n}_1 + g(H)]^{-2} > \left\{ \left(\frac{\beta \bar{n}_1}{2} \right)^2 g^{-3}(H) \right\} [\bar{n}_1 + g(H)] \quad (89)$$

and from (86):

$$\theta + \left(\frac{\beta \bar{n}_1}{2} \right)^2 g^{-2}(H) > \bar{n}_1 \left(\frac{\beta \bar{n}_1}{2} \right)^2 g^{-3}(H) + \left(\frac{\beta \bar{n}_1}{2} \right)^2 g^{-2}(H). \quad (90)$$

Therefore: $\frac{dn_2^*}{dH} > 0$ iff : $\theta > \bar{n}_1 \left(\frac{\beta \bar{n}_1}{2} \right)^2 g^{-3}(H)$ i.e. iff $\theta > \frac{\beta^2}{4} \left(\frac{\bar{n}_1}{n_2^*} \right)^3$. From second order necessary condition (85), this condition is necessarily fulfilled.

In Figure 6 we show how n_2^* varies with H , for $n_1 = 10, \beta = 0.10$ and $\theta = 0.20$. We apply $H \geq 7$, as we numerically verify that for $H < 7$ the employer prefers the type 1 only employment regime.

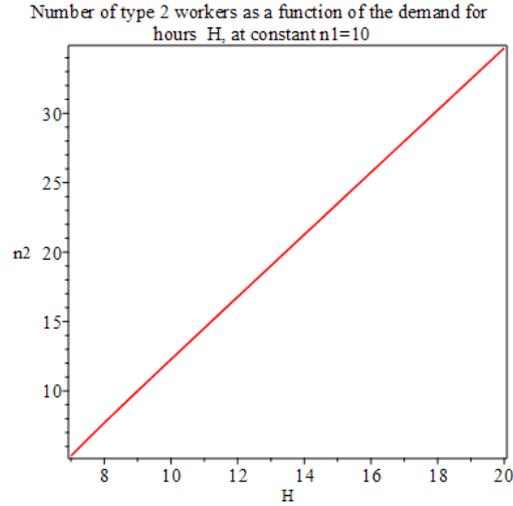


Figure 6: Demand for type 2 workers, depending on H

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