Working time and wage rate differences : a contract theory approach

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Abstract

In the labor economics literature, discrimination is often defined as a situation in which identically productive workers, placed in the same working conditions, are treated unequally, being assigned contracts involving in particular different hourly wage rates. In the proposed analysis, the contract theory approach is applied, contributing to explain how in some circumstances such differences take place, even if contract discrimination and productivity differences are strictly ruled out. It is assumed that workers types differ only in their leisure consumption preferences and in their availability. A labor cost-minimizing firm offers a menu of labor contracts, and let workers self-select. In this non-discriminating setting the model reveals the possibility of a paradoxical situation in which the less demanding workers obtain a higher wage rate. It brings out external effects between types and the existence of a quantum (a minimum number) of demanded workers for some type.

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1 Introduction

According to the standard labour market theory, wage rates tend to reflect marginal productivity of labour services. Consequently, they should coincide for workers performing the same task with the same ability. Labour market observations however, in many instances, tend to challenge this view. To explain the differences between the neoclassical expected equality and facts, the relevant literature has often resorted to the notion of discrimination. Labor market discrimination has been defined as "a situation in which persons who provide labor market services and who are equally productive in a psychical or material sense are treated unequally in a way that is related to an observable characteristic such as race, ethnicity, or gender" (Altonji and Blank, 1999).[1]

Genuine discrimination can take place at least under two independent sufficient conditions: if prejudices or stereotypes about productivity for different profiles of workers prevail, employers make suboptimal contracting decisions, based on biased estimates and their discriminating policy is in fact detrimental to their own interest. Another more perverse origin could be found when employers, although possibly not biased in their perceptions of the workers abilities have a non conventional conduct, discrimination itself, for instance some gender supremacy, being included in their own objective (Cain, 1986; Altonji et Blank, 1999; Cahuc et al., 2014). Such behaviors, consistently driving a firm out of its profit maximization track, are hardly compatible with intense competition (Becker, 1957), but could be sustainable in monopsonistic conditions and in markets with trade frictions (e.g., Borjas and Bronars, 1989; Black 1995; Barth and Dale-Olesen, 2009). Bertrand et al. (2010) show that the gender gap among graduates of a single prestigious MBA increases with seniority; this trend can be explained almost entirely by differences in hours worked due to a combination of women working fewer hours per week, conditional on working, and also being more likely to have gaps in their careers which can, in turn, be explained by child rearing (OECD, 2018). In such cases, actual productivity is "path dependent" and is not perfectly captured by standard proxies mainly related to education. Compensation differences in such sit-

[1] One discrimination topic that received significant attention in the last few years, is the gender wage gap. Despite convergence in educational attainments (in the US) and in other relevant explanatory variables, unexplained differences still exist between male and female wages, and particularly so at the top of the wage distribution (Gunderson, 1989; Blau and Kahn, 2017; Cahuc et al., 2014; OECD, 2018; Neumark, 2018).

uations do not reflect a particular failure of the labour market, but a more encompassing feature of the social system.

It is well known however, that some observed differences in wages obtained by equally competent workers may be explained without resorting to discrimination. The theory of compensating differentials, originating in Adam Smith's celebrated seminal works, argues that wage gaps can be grounded in differences in preferences for job attributes (Thaler and Rosen, 1975; Rosen, 1986).[2] Some other explanations recede in fact to a productivity argument: in her Presidential address, Goldin (2014) pointed out that in some qualified occupations (legal services, business, finance) a convex hours-earnings relationship prevails, longer hours as provided by men in general, being rewarded at a higher wage rate.[3] She explains this outcome by productivity increasing in hours, as firms and their clients value "temporal flexibility": on-site presence, intensive client contact, face-to-face time, etc. If men are more available than women for such jobs, then a gender wage gap could result.

Our text aims at providing a particular theoretical interpretation of the stylized facts mentioned by this literature in introducing a model of paying pattern determination completely free of productivity differences and in which all workers of different types have equal access to all contracts proposed by the employer. Our model contributes to show how this "ex-ante" non-discrimination may be compatible with "ex-post" discrimination, conceived as different prices paid for identical labour services. Ex-ante non-discrimination is interpreted as the impossibility for the employer to assign specific contracts to the various types. This equal access of the employees to the full set of contracts may be a consequence of either efficient legislation or of the simple impossibility of distinguishing types. Therefore, optimal contracts are confronted with participation and incentive compatibility constraints as emphasized in contract theory (inter alia, Bolton and Dewatripont, 2005; Salanié, 2005; Laffont and Martimort, 2009), each type freely choosing the contract that

[2] For a recent illustration, Cook and al. (2018) study the wages earned by Uber drivers, and detect a 7% gap in favor men. Since longer working schedules apparently have in this case little impact on the wage rate, it is suggested that male drivers earn a compensating differential for their willingness to drive in areas with higher crime rates and more drinking establishments. It must be noticed that in such a case, male and female labour services are not perfect substitutes.

[3] See also Goldin (2015). Cortés and Pan (2019) bring empirical evidence documenting a positive relationship between the demand for long hours and the wage gap for high-skilled employees, while Denning et al. (2019) find no such relationship.

was designed for him/her. In monopoly pricing theory, the situation in which consumers with unobservable preferences have access to all available price-quantity bundles posted by the firm is analyzed within the framework of non-linear pricing (or second-degree price discrimination).[4]

To keep the analysis as simple as possible, we consider that workers are of only two types, workers of the first type always demanding less compensation for any given working time than workers of the second type. The firm in our analysis needs a predetermined amount of hours to achieve its production target. The total labour cost includes worker compensation and a fixed cost per worker (Rosen, 1968; Hart, 1987; Contensou and Vranceanu, 2001). The goal of the firm is to minimize the cost of hiring the required amount of hours.[5] In such a case, if workers of both types are available in any quantity, cost minimization would generally induce labour demand to concentrate on the "cheaper" type; the more demanding workers being crowded out, they do not appear in data. But if workers of the first type are in scarce supply, the employer may be induced to hire workers of both types.

In this case, if contract discrimination is allowed (the firm assigning a different contracts to each type), then it is shown that the cost minimizing contracts may imply lower wage rates and longer working hours for the less demanding type. However, and this is the key point of our analysis, if contract discrimination is not possible, hiring available workers of the more demanding type creates an externality influencing the contract offered to the less demanding type, through an incentive compatibility constraint. Our text aims at solving this more involved problem.

The main consequences of the proposed analysis are:

a) It is shown that ordering the worker types by non-crossing compensation functions is not sufficient to predict the ordering in optimally contracted working times, compensations and implicit hourly wages. Therefore the model makes possible to bring out the crucial role played by specific assumptions concerning the sensibility of the worker compensation differential with respect to working time itself. Differences between compensation functions while constant in sign, may be time increasing, constant or time decreasing. It the difference is time increasing, the less

[4] See for surveys of this literature: Varian (1986), Wilson (1993) or Armstrong (2016).

[5] This setting is not a situation of monopsony hiring labor. While the latter decides on the labor contract taking into account its market power in the labor market, in our framework the firm has no market power.

demanding workers do longer hours, like in the discriminating case; however, they will do less hours under the opposite assumption.

b) The analysis reveals the fact that the employer minimizes labour cost by offering contracts generally exhibiting different (implicit) hourly wages. We refer to these wage differences as "pseudo-discrimination", since these wage-rates differences between workers doing the same job with the same craft exist, but are not explained by wrongly perceived productivity differences or by a biased objective function of the employer. In particular, the model reveals the possibility of a paradoxical situation in which the less demanding workers are granted a higher wage rate, in stark contrast with the case in which discrimination is possible.

c) Finally, in formalizing and interpreting the labour cost function, the model explains the demand for the more exacting workers and predicts local discontinuity. For some threshold in needed working services, the cost minimizing policy switches from employing the less demanding workers only, to a mixed labour force, including a minimum number (quantum) of the more demanding type. This is compatible with some form of mass redundancy in case of economic slowdown.

Our analysis is abstract and does not address gender discrimination in particular, but can include it as a superimposed interpretation. Indeed, consumption/leisure preferences can be specific to the gender of the employee. In this case, the model brings out possible gender wage differentials in the absence of any biased information about their productivity or of non-conventional objective of the employer.

The paper is organized as follows: In Section 2, we introduce the main assumptions and define the possible mixed employment regimes. Section 3 analyses the cost minimization problem in the general case. In section 4, we provide a more precise definition of the optimal contracts introducing a more specific preference structure, the compensation differential being a linearly increasing function in working hours. In section 5 more precision is obtained from adopting a quadratic compensation functions hypothesis and numerical simulations are used to buttress analytical results. The minimum number of type 2 workers is made explicit from parameters. Section 6 is our conclusion.

2 The model

2.1 General assumptions

We analyze the cost minimizing labor contracts designed by an employer who needs a given amount of labour services H per time unit.

Homogeneous working services measured by hours

Technology is such that H can be expressed by a sum of hours considered as perfect substitutes in production. The objective of the employer consists in minimizing the total cost of the H hours. Working hours are not only homogeneous in terms of productivity but also in terms of working conditions, as applying to the same task. Differences related to the intrinsic disutility of work, as considered by the theory of compensating differentials, are therefore ruled out.

Two types of workers:

There are two types of workers, $i = (1, 2)$, demanding different compensations for a given working time. The utility function of the worker i is represented by $s = u_i(c, h)$, where c is the obtained purchasing power and h stands for hours worked in the same time interval. Identity $\bar{s}_i \equiv u_i\left[c, v_i(h)\right]$ defines $v_i(h)$ the *compensation function*, indicating the minimum compensation required by type i individual to supply h hours of work. The participation constraint for type i is thus satisfied if $c \geq v_i(h)$.

In line with the standard neoclassical assumptions, the function $v_i(h)$ is supposed convex $v_i''(h) > 0$ and such that $v_i(0) \geq 0$, $v_i'(h) > 0$.[6]

Ordered compensation requirements

We assume that type 2 workers are always demanding *more* compensation than type 1 workers for any given working time :

$$v_2(h) > v_1(h), \ \forall h > 0. \tag{1}$$

The two types are consequently unequivocally ordered in this respect, *the two compensation functions do not cross.*

Fixed non-wage cost per worker

[6] If $v_i(0) > 0$, such minimal compensation may be interpreted as an indivisible cost of the job for the worker, such as having to commute to the working place.

The cost of labour includes a per individual, fixed, non-wage expense (cost of a working place, non related to worked hours), denoted by $\theta > 0$ (Rosen, 1968; Hart, 1987; Contensou and Vranceanu, 2001). This fixed cost is supposed independent upon type i.

Non-discrimination assumption as free access of all workers to all contracts

The employer proposes two labor contracts $P_1 = (h_1, c_1)$ and $P_2 = (h_2, c_2)$ determining payment c_i and hours h_i.

A basic assumption is that *the employer cannot discriminate*, i.e., cannot prevent workers from choosing their preferred contract. This can be interpreted as a consequence of either legal or social constraints banning discrimination if types can be observed, or just of imperfect information, the employer not being able to distinguish the types.

Scarcity of type 1 workers

Type 2 workers are abundant, i.e. they are available in any number n_2. The less demanding type of workers (type 1) is in limited supply. Let $\bar{n}_1$ denote the number of type 1 workers available in the labour market, $n_1 \leq \bar{n}_1$ and therefore cost minimization may induce the employer to hire simultaneously type 2 workers in order to keep with its production objective.

Sufficiency or scarcity of the less demanding workers, pure and mixed regimes

If the number H of needed hours is low enough, the employer could hire only the less demanding type 1 workers. Hiring both types will take place if the demand for hours is high enough. A comparison of the cost functions (minimum costs) associated to different hiring possibilities will allow us to present the conditions in which the non-trivial case of a mixed employment regime can prevail; we first formalize the cost minimizing decision in the simple case when one type is employed.

2.2 Cost functions working with one type exclusively

We first consider the elementary problem in which cost is minimized by hiring only one type of workers, supposed available in any number.

Minimizing the cost of H hours *with one type of workers only*, under the saturated participation constraint $c_i = v_i(h_i)$ and with the per worker fixed cost θ is equivalent to minimizing:

$$C_i(H) = n_i\left[v_i(h_i) + \theta\right], \text{ with } n_i h_i = H, \text{ for } i = (1, 2). \tag{2}$$

The first order condition implies:

$$v_i'(\hat{h}_i) = \frac{v_i(\hat{h}_i) + \theta}{\hat{h}_i}, \tag{3}$$

where $\hat{h}_i$ is the first-best, or *notional working time.* We show in Appendix 1 that the solution $\hat{h}$ to equation (3) is unique.[7] Equation (3) reflects equality of marginal and average cost of hours.

The corresponding *notional employment* is $\hat{n}_i = \dfrac{H}{\hat{h}_i}$ workers.

The resulting minimized cost function noted $\hat{C}_i(H)$ is therefore linear in H:

$$\hat{C}_i(H) = H\frac{v_i(\hat{h}_i) + \theta}{\hat{h}_i} = Hv_i'(\hat{h}_i). \tag{4}$$

The two different cost functions (4) for type 1 or type 2 workers independently employed are clearly ordered by assumption (1) and, $\forall H > 0$:

$$\hat{C}_1(H) = H\frac{v_1(\hat{h}_1) + \theta}{\hat{h}_1} < H\frac{v_2(\hat{h}_2) + \theta}{\hat{h}_2} = \hat{C}_2(H). \tag{5}$$

To formally prove (5) we write: $\dfrac{v_2(\hat{h}_2) + \theta}{\hat{h}_2} > \dfrac{v_1(\hat{h}_2) + \theta}{\hat{h}_2}$ (by assumption 1) and $\dfrac{v_1(\hat{h}_2) + \theta}{\hat{h}_2} > \dfrac{v_1(\hat{h}_1) + \theta}{\hat{h}_1}$ (from uniqueness of notional working time $\hat{h}_1$).

Obviously, as long as type 1 workers are available, the firm would hire them only and not the more demanding type 2 workers.

For $\hat{n}_1 = \dfrac{H}{\hat{h}_1} \leq \bar{n}_1$ the minimized labour cost function is:

$$\hat{C}(H) = \hat{C}_1(H) = H\frac{v_1(\hat{h}_1) + \theta}{\hat{h}_1} = Hv_1'(\hat{h}_1). \tag{6}$$

As long as $\hat{n}_1 \leq \bar{n}_1$, the firm would hire and attract only type 1 workers by posting the contract $\hat{P}_1 = \left[\hat{h}_1, v_1(\hat{h}_1)\right]$. From the limit $\bar{n}_1$, the validity domain of (6) is bounded above by

$$H \leq H_1 = \bar{n}_1\hat{h}_1 \tag{7}$$

[7] For instance, with a quadratic compensation function $\nu(h) = h^2$, the notional working time is $\hat{h} = \sqrt{\theta}$, and the notional employment is $\hat{n} = H/\sqrt{\theta}$.

For $H \geq H_1$, the employer must compensate the type 1 workers for longer hours, and since $h_1 = \dfrac{H}{\bar{n}_1} > \hat{h}_1$, the relevant cost function becomes:

$$C_1(H) = \bar{n}_1 \left[v_1 \left(\frac{H}{\bar{n}_1} \right) + \theta \right], \tag{8}$$

an increasing and convex function of H.

2.3 Cost minimization employing both types

For some value of H, the employer can minimize cost, switching from type 1 not to type 2 exclusive employment, but to a mixed regime, hiring type 2 workers and still taking advantage of the less demanding type 1 workers.

If contract discrimination is feasible, Appendix 2 solves an example (with quadratic compensation functions) in which the cost minimizing contracts involve longer working hours and a *lower wage rate* for the less demanding type.

Without contract discrimination, the behavior of the cost minimizing employer is different. Indeed, introducing type 2 workers with contract P_2 has two opposite effects on the determining elements of labour cost:

- on the first hand, it enables to reduce type 1 working time h_1 and its compensation.

- on the other hand, discrimination being ruled out, opening a P_2 contract introduces a costly new constraint on contract P_1, which must be at least as attractive as P_2 for type 1 workers. This supplementary constraint is in fact a "fixed cost" which must be paid whatever small the number of hired type 2 workers, suggesting the possible necessity of hiring simultaneously a minimum amount of type 2 workers.

Opening a contract P_2 in order to minimize labour cost is justified by H when the optimal choice of contracts P_1 and P_2, i.e., P_1^* and P_2^*, induces a labour cost $C^*(H)$ smaller than $C_1(H)$ as defined in expression (8). The switching value of H is noted $\check{H}$.

This brings out the necessity of understanding optimal contract determination, employing the two types simultaneously.

In this case, the employer has to post two contracts $P_i = (c_i, h_i)$ for $i = (1, 2)$ and determine the number n_2 of hired type 2 workers. The cost to be minimized is:

$$C(H) = n_1(c_1 + \theta) + n_2(c_2 + \theta), \tag{9}$$

the control variables being $\{c_1, h_1, c_2, h_2, n_1, n_2\}$, with $n_1 \leq \bar{n}_1$, with participation and incentive compatibility constraints for the two types, and with the quantity constraint:

$$n_1 h_1 + n_2 h_2 = H. \tag{10}$$

The solution $(c_1^*, h_1^*, c_2^*, h_2^*, n_1^*, n_2^*)$ entails a cost function:

$$C^*(H) = n_1^*(c_1^* + \theta) + n_2^*(c_2^* + \theta). \tag{11}$$

As will be illustrated in the following (section 4.3), a switch from the type 1 exclusive employment to the mixed labour force regime is optimal only in introducing a *quantum*, i.e., a minimum number of type 2 workers.

As an upshot of this cost analysis, H_1 being defined in (7) the cost function $C(H)$ takes the form:

$$C(H) = \begin{cases} \hat{C}_1(H) = Hv_1'(\hat{h}_1) & \text{for } 0 < H \leq H_1 \quad \text{(Type 1 only, notional hours)} \\ C_1(H) = \bar{n}_1 \left[v_1\left(\dfrac{H}{\bar{n}_1}\right) + \theta \right] & \text{for } H_1 < H \leq \check{H} \quad \text{(Type 1 only, longer hours)} \\ C^*(H) = n_1^*(c_1^* + \theta) + n_2^*(c_2^* + \theta). & \text{for } \check{H} < H. \quad \text{(Non-discriminating mixed regime)} \end{cases} \tag{12}$$

In the following, the analysis will focus on the non-trivial case in which *the demand for hours is large enough ($H > \check{H}$) to justify the non discriminating mixed regime.*

2.4 The non-discriminating mixed regime

We follow the standard resolution steps taken in contract theory analysis (Laffont and Martimort, 2001); we first determine the feasible allocations taking into consideration both participation and incentive compatibility constraints, then analyze the firm's optimization problem (cost minimization here) within the set of feasible allocations.

2.4.1 Participation and incentive compatibility constraints: first consequences

If both types are employed, taking the hour constraint (10) into consideration, total cost minimization *does not* consist in minimizing independently the cost of hours supplied by the two types since existence of a (c_i, h_i) contract modifies the terms of the incentive compatibility constraint applying to type j workers. The set of constraints comprises:

1) Participation constraints:

$$c_1 \geq v_1(h_1) \quad (13)$$

$$c_2 \geq v_2(h_2) \quad (14)$$

2) Incentive compatibility constraints:

Type 1 workers being eligible to contract $P_2 = (c_2, h_2)$, they prefer or accept the contract $P_1 = (c_1, h_1)$ only if:

$$c_1 - v_1(h_1) \geq c_2 - v_1(h_2) \quad (15)$$

Similarly, type 2 prefer or accept the contract P_2 to contract P_1 only if:

$$c_2 - v_2(h_2) \geq c_1 - v_2(h_1) \quad (16)$$

Consequently, we can state a first property of the contracts compatible with the two types of workers.

Proposition 1 *If compensation differences* $[v_2(h) - v_1(h)]$ *are time increasing, then* $h_1 \geq h_2$*; if they are time decreasing, then* $h_2 \leq h_1$.

Proof. Adding (15) and (16) :

$$v_2(h_1) - v_1(h_1) \geq v_2(h_2) - v_1(h_2). \quad (17)$$

If compensation differences $[v_2(h) - v_1(h)]$ are increasing in h, the inequality (17) is not compatible with $h_1 < h_2$. If they are decreasing in h, the inequality (17) is not compatible with $h_1 > h_2$. ■

This rule being a consequence of the incentive constraints, it applies not only to optimum values of working hours (cost minimizing values), but also to all feasible values.

In our context, the general set of constraints admits particular properties. We emphasize here the redundancy of the system of constraints, as well as the necessary and unnecessary surpluses.

1) From the IC condition (15), $c_1 - v_1(h_1) \geq c_2 - v_1(h_2)$. Participation condition for type 2 (14) requires $c_2 \geq v_2(h_2)$. Thus $c_1 - v_1(h_1) \geq v_2(h_2) - v_1(h_2)$, and from our basic assumption (1)

$v_2(h_2) - v_1(h_2) > 0$. Consequently:

$$c_1 - v_1(h_1) \geq v_2(h_2) - v_1(h_2) > 0.$$

This makes the participation constraint for the type 1 redundant and indicates the necessity *of a positive surplus* $v_2(h_2) - v_1(h_2)$ for the *type 1.*

2) Type 2 workers being supposed available in any quantity, they obtain no surplus in the solution; their participation constraint must be saturated:

$$c_2 = v_2(h_2). \tag{18}$$

Limiting c_2 to its minimum value $v_2(h_2)$ decreases the cost of workers of type 2 directly, and the cost of workers of type 1 indirectly, by mitigating the cost of the relevant incentive compatibility constraint.

If $c_2 = v_2(h_2)$, the incentive compatibility constraint for type 1 (condition 15) becomes :

$$c_1 - v_1(h_1) \geq v_2(h_2) - v_1(h_2). \tag{19}$$

In this expression, $v_2(h_2) - v_1(h_2)$ is the surplus obtained by type 1 choosing contract P_2. Therefore:

3) In its saturated form , the type 1 incentive compatibility constraint implies:

$$c_1 = v_1(h_1) + v_2(h_2) - v_1(h_2). \tag{20}$$

4) It must be noticed that in our assumptions, the incentive compatibility constraint for type 2 is satisfied if type 2 workers have no surplus in accepting contract $P_1 = [c_1, h_1]$. This condition is fulfilled if: $c_2 - v_2(h_2) = 0 \geq c_1 - v_2(h_1)$ for $c_1 = v_1(h_1) + v_2(h_2) - v_1(h_2)$, i.e., $v_1(h_1) + v_2(h_2) - v_1(h_2) - v_2(h_1) \leq 0$ or $v_2(h_1) - v_1(h_1) \geq v_2(h_2) - v_1(h_2)$.

From the rule induced by Proposition 1 , inequality (17) holds and (18), (20) imply (16). This last result will enable us to omit explicit treatment of condition (16) in the constrained cost minimization problem.

2.4.2 Cost minimization: first order conditions

Let $C(n_1, n_2, h_1, h_2, H)$ stand for the explicit form of the cost function $C(H)$:

$$C(n_1, n_2, h_1, h_2, H) = n_1(c_1 + \theta) + n_2(c_2 + \theta). \tag{21}$$

We apply the substitutions introduced in (18) and (20), and obtain the cost function:

$$C(n_1, n_2, h_1, h_2, H) = n_1 \left[v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta\right] + n_2 \left[v_2(h_2) + \theta\right]. \tag{22}$$

The total hours constraint is:

$$n_1 h_1 + n_2 h_2 - H = 0. \tag{23}$$

Limited supply of type 1 workers imposes $n_1 \leq \bar{n}_1$. We show (Appendix 3) that this constraint is necessarily binding in our assumptions ($H > \check{H}$), and substitute n_1 with $\bar{n}_1$.

The corresponding Lagrangian is:

$$L = \bar{n}_1 \left[v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta\right] + n_2 \left[v_2(h_2) + \theta\right] - \lambda \left[\bar{n}_1 h_1 + n_2 h_2 - H\right] \tag{24}$$

First-order conditions applying to interior solutions are:

$$\frac{\partial L}{\partial n_2} = v_2(h_2^*) + \theta - \lambda h_2^* = 0 \tag{25}$$

$$\frac{\partial L}{\partial h_1} = \bar{n}_1 v_1'(h_1^*) - \lambda \bar{n}_1 = 0 \tag{26}$$

$$\frac{\partial L}{\partial h_2} = \bar{n}_1 \left[v_2'(h_2^*) - v_1'(h_2^*)\right] + n_2^* v_2'(h_2^*) - \lambda n_2^* = 0 \tag{27}$$

$$\frac{\partial L}{\partial \lambda} = \bar{n}_1 h_1^* + n_2^* h_2^* - H = 0 \tag{28}$$

Once the system solved for $(\lambda^*, n_2^*, h_1^*, h_2^*)$, the compensations associated to each contract are $c_1^* = v_1(h_1^*) + v_2(h_2^*) - v_1(h_2^*)$, and $c_2^* = v_2(h_2^*)$.

We can further notice that condition (26) implies:

$$v_1'(h_1^*) = \lambda. \tag{29}$$

Thus conditions (25) and (26) imply:

$$\frac{v_2(h_2^*) + \theta}{h_2^*} = v_1'(h_1^*). \tag{30}$$

Equation (30) indicates the equality of the marginal cost of hours obtained from the two possible sources: increasing the number of type 2 workers at constant working time or increasing the working time of type 1 workers in constant number.

Finally, conditions (26) and (27) imply:

$$\bar{n}_1 \left[v_2'(h_2^*) - v_1'(h_2^*)\right] + n_2^* \left[v_2'(h_2^*) - v_1'(h_1^*)\right] = 0. \tag{31}$$

The corresponding labour cost function is:

$$C^*(H) = C(n_2^*, h_1^*, h_2^*, H), \text{ with } H > \check{H}. \tag{32}$$

It can be checked, after relevant substitutions, that the Lagrangian multiplier λ^* reflects marginal cost of hours, from type 1 or from type 2.

$$\frac{dC^*(H)}{dH} = \lambda^* = v_1'(h_1^*) = \frac{v_2(h_2^*) + \theta}{h_2^*}. \tag{33}$$

2.4.3 Optimal hours and the structure of preferences

Proposition 1 revealed that the properties of the contracts (the hours ordering) depend in a significant way on whether the difference $[v_2(h) - v_1(h)]$ is increasing, constant or decreasing in h.

A time increasing difference $[v_2(h) - v_1(h)] \Leftrightarrow v_2'(h) - v_1'(h) > 0$, $\forall h$. In this case, Proposition 1 implies that $h_1 > h_2$, regardless on whether working hours are optimal or not. Thus:

Proposition 2 *If difference in demanded compensations is time increasing, type 2 workers are assigned a shorter working time than type 1 workers, i.e.* $h_2^* < h_1^*$.

Also,

Proposition 3 *If difference in demanded compensations is constant, type 1 and type 2 workers are assigned the same working time, corresponding to the notional value i.e.* $h_2^* = h_1^* = \hat{h}_2 < \hat{h}_1$

Proof. Proof: in this case, $v_2'(h) = v_1'(h)$ and equation (27) $\bar{n}_1 [v_2'(h_2^*) - v_1'(h_2^*)] + n_2^* v_2'(h_2^*) - \lambda n_2^* = 0$ may be written: $0 + n_2^* [v_2'(h_2^*) - v_1'(h_1^*)] = n_2^* [v_1'(h_2^*) - v_1'(h_1^*)] \Rightarrow h_1^* = h_2^*$. ■

3 The case of linearly increasing difference in compensations

We suggest in the following to scrutinize the consequences of the following simple preference structure:

$$v_2(h) = v_1(h) + \beta h, \tag{34}$$

where β is a positive constant. Under this assumption, not only the type 2 worker is more demanding than the type 1 (see condition 1), but the difference $[v_2(h) - v_1(h)]$ is *linearly* increasing in h.

3.1 Coincidence of notional working times

We first notice that in this case, the notional working time $\hat{h}$ defined by equation (3) i.e., the working time the employer would choose if the contracts were independently determined thanks to discrimination, is the same for the two types:

$$\hat{h}_1 = \hat{h}_2 \equiv \hat{h} \tag{35}$$

We first notice that equation (34) implies $v_2'(h) = v_1'(h) + \beta$. It can be checked that if $v_1'(h) = \dfrac{v_1(h)+\theta}{h}$, $v_2'(h) = \dfrac{v_2(h)+\theta}{h}$ implying: $h = \hat{h} = \hat{h}_2 = \hat{h}_1$.

It is now possible to predict the ordering of contracted working times in the non-discriminating case.

3.2 Working time ordering

Proposition 4 *Non-discriminating optimal policy shifts the working times included in the two contracts from their common notional value* $\hat{h}$ *in opposite directions:* $h_2^* < \hat{h} < h_1^*$.

Proof. From assumption 34: $v_2'(h_2^*) = v_1'(h_2^*) + \beta$. Then condition (31) becomes:

$$\bar{n}_1\beta + n_2^*\left[v_1'(h_2^*) - v_1'(h_1^*) + \beta\right] = 0 \tag{36}$$

or equivalently:

$$v_1'(h_1^*) - v_1'(h_2^*) = \beta\frac{(\bar{n}_1 + n_2^*)}{n_2^*} \tag{37}$$

Since $\beta > 0$, then $v_1'(h_1^*) > v_1'(h_2^*)$. From strict convexity of $v_1(h)$, we have $v_1'(h_1^*) > v_1'(h_2^*) \Rightarrow h_1^* > h_2^*$. ■

We can further compare these working times with the notional common working time, $\hat{h}$, and prove the following inequality:

$$h_2^* < \hat{h} < h_1^*. \tag{38}$$

We first demonstrate that $h_1^* > \hat{h}$.

We start from the definition of the notional working time $\hat{h}_2 = \arg\min\left[\dfrac{v_2(h)+\theta}{h}\right]$ and use equality (35) $\hat{h}_1 = \hat{h}_2 = \hat{h}$. Uniqueness of $\hat{h}_2$ (Appendix 1) implies:

$$\frac{v_2(h_2^*)+\theta}{h_2^*} \geq \frac{v_2(\hat{h}_2)+\theta}{\hat{h}_2} = v_2'(\hat{h}_2). \tag{39}$$

But from equation (30), $\dfrac{v_2(h_2^*)+\theta}{h_2^*} = v_1'(h_1^*)$ and inequality (39) implies $v_1'(h_1^*) \geq v_2'(\hat{h}_2)$.

Since $\hat{h}_1 = \hat{h}_2 = \hat{h}$ and $v_2'(h) = v_1'(h) + \beta$, equation (39) implies $v_1'(h_1^*) > v_1'(\hat{h})$ and (from strict convexity), $h_1^* > \hat{h}$.

We next demonstrate that $h_2^* < \hat{h}$.

From equation (37), $v_1'(h_2^*) = v_1'(h_1^*) - \dfrac{(\bar{n}_1 + n_2^*)}{n_2^*}\beta$.

By assumption, $v_2(h) = v_1(h) + \beta h$, and from equation (30): $\dfrac{v_1(h_2^*)+\theta}{h_2^*} + \beta = v_1'(h_1^*)$. Then equation (37) implies:

$$v_1'(h_2^*) = \frac{v_1(h_2^*)+\theta}{h_2^*} + \beta - \frac{(\bar{n}_1 + n_2^*)}{n_2^*}\beta. \tag{40}$$

Therefore, $v_1'(h_2^*) < \dfrac{v_1(h_2^*)+\theta}{h_2^*}$.

Considering simultaneously $v_1'(h_2^*) < \dfrac{v_1(h_2^*)+\theta}{h_2^*}$ and $v_1'(\hat{h}) = \dfrac{v_1(\hat{h})+\theta}{\hat{h}}$, implying:

$$v_1(h_2^*) - h_2^* v_1'(h_2^*) > -\theta \text{ and } v_1(\hat{h}) - \hat{h} v_1'(\hat{h}) = -\theta. \tag{41}$$

As shown in the Appendix 1 (see equation 60), the function $\varphi(h) \equiv v(h) - hv'(h)$ is monotonously decreasing, thus equation (41) implies: $h_2^* < \hat{h}$.

In this solution the employer is "deteriorating" on purpose the terms of the contract offered to the most demanding type, to cut down the surplus offered to the least demanding type, along the lines of the classic damaged goods analysis by Deneckere and McAfee (1996).

3.3 The consequence of increasing demand for hours

From necessary first order conditions applying to (24), it is possible to predict the effect of indefinitely increasing needed hours on optimally contracted working times.

Proposition 5 *When the total demand for hours H indefinitely increases, for constant $\bar{n}_1$, hours h_2^* in the optimal contract P_2^* tend to the notional value $\hat{h}$.*

Proof. From equation (23), we notice that when the demand for hours increases ($H \to \infty$), since $n_1 \leq \bar{n}_1$ is fixed and since h_1 and h_2 cannot increase indefinitely, necessarily $n_2 \to \infty$. From equation (37) $v_1'(h_1^*) - v_1'(h_2^*) = \beta\dfrac{(\bar{n}_1 + n_2^*)}{n_2^*}$. If $n_2^* \to \infty$, then $[v_1'(h_1^*) - v_1'(h_2^*)] \to \beta$.

From equation (30), $\dfrac{v_2(h_2^*)+\theta}{h_2^*} = v_1'(h_1^*)$ and since by assumption, $v_2(h_2^*) = v_1(h_2^*) + \beta h_2^*$, $\dfrac{v_2(h_2^*)+\theta}{h_2^*} = \dfrac{v_1(h_2^*)+\beta h_2^*+\theta}{h_2^*} = \dfrac{v_1(h_2^*)+\theta}{h_2^*} + \beta = v_1'(h_1^*)$. Since $v_1'(h_1^*) \to v_1'(h_2^*) + \beta$, equation

(30) implies: $\dfrac{v_1(h_2^*)+\theta}{h_2^*} \to v_1'(h_2^*)$. The limit of this equation is only compatible (uniqueness) with the notional cost minimizing working time for type 1 workers (and for type 2 in our special assumption). Therefore,

$$H \to \infty \Rightarrow n_2^* \to \infty \Rightarrow h_2^* \to \hat{h}_2 = \hat{h}. \tag{42}$$

■

As intuitively predictable, when the proportion of type 2 workers indefinitely increases, the external influence on their contract by type 1 preferences dwindles, type 2 working time tends towards its notional optimal value.

Proposition 6 *When the total demand for hours H indefinitely increases, hours h_1^* in optimal contract P_1 always exceed the notional hours $\hat{h}$.*

Proof. With respect to the evolution of contract P_1^* we notice that from equation (37) $[v_1'(h_1^*) - v_1'(h_2^*)] \to \beta$ and simultaneously, $h_2^* \to \hat{h} = \hat{h}_1$ implying: $v_1'(h_1^*) \to v_1'(\hat{h}) + \beta$ and therefore $h_1^* > \hat{h}$. ■

The working time for type 1 is kept above its notional value under the influence of β, i.e., this parameter standing for the higher sensibility of type 2 with respect to hours.

3.4 Ordering wage rates

The first order conditions (25) to (28) are a priori compatible with any wage rate ordering.

The wage rate of the type 2 worker is $w_2 = c_2^*/h_2^*$, and, since no surplus is needed for the participation of type 2, $c_2^* = v_2(h_2^*)$. The wage rate can be written $w_2 = v_2(h_2^*)/h_2^*$. From (30), $\dfrac{v_2(h_2^*)+\theta}{h_2^*} = v_1'(h_1^*) \Leftrightarrow \dfrac{v_2(h_2^*)}{h_2^*} = v_1'(h_1^*) - \dfrac{\theta}{h_2^*}$. Thus:

$$w_2 = v_1'(h_1^*) - \frac{\theta}{h_2^*}. \tag{43}$$

The wage rate of the type 1 is $w_1 = c_1^*/h_1^*$, where according to IC constraint (20), $c_1^* = v_1(h_1^*) + v_2(h_2^*) - v_1(h_2^*)$. With $v_2(h) = v_1(h) + \beta h$, the compensation of type 1 workers is $c_1^* = v_1(h_1^*) + \beta h_2^*$. The wage rate is:

$$w_1 = \frac{v_1(h_1^*) + \beta h_2^*}{h_1^*} = \frac{v_1(h_1^*)}{h_1^*} + \beta \frac{h_2^*}{h_1^*}. \tag{44}$$

Therefore:

$$w_1 \lesseqgtr w_2 \text{ if } \theta \frac{h_1^*}{h_2^*} + \beta h_2^* \lesseqgtr v_1(h_1^*)\left[\eta(h_1^*) - 1\right] \text{ with } \eta(h_1^*) = \frac{h_1^* v_1'(h_1^*)}{v_1(h_1^*)}. \tag{45}$$

As mentioned in the introduction, several scholars (Goldin, 2014; Cortés and Pan, 2019) brought evidence according to which in some high-skilled occupations long hours of work are associated with higher hourly wages, and explained this difference by the higher productivity of the longer hours, defined as a better service to clients and customers who require "temporal flexibility".

Combining contract theory and asymmetric quantitative limits on labour services supply, our model proposes an alternative explanation to such facts. Indeed, depending on the parameters, it is possible to show that the cost minimizing contracts sometimes provide a *higher wage rate* to the *less demanding type 1* workers (who work longer hours) not resorting to any productivity argument.

In order to bring out the possibility and the necessary conditions for this "paradoxical" wage rate ordering, and to obtain more details with employers behavior, we introduce the supplementary assumption quadratic utility compensation function.

4 The case of quadratic compensation functions

The specific compensated consumption structure used before allowed us to better characterize the optimal hours in contracts P_1 and P_2 when compensation differences are time increasing. Introducing a even more precise structure of the compensation functions enables to entirely solve the model for the endogenous variables and reveal a possible paradoxical ordering of hourly wage rates. We will also resort to a numerical simulation to provide additional intuition about the properties of the solution.

The quadratic function $v(h) = h^2$ has the required property of strict convexity and involves existence of a notional working time. Our compensating consumption functions become:

$$v_1(h) = h^2 \text{ and consequently } v_2(h) = h^2 + \beta h. \tag{46}$$

4.1 The solution to the cost minimization problem

4.1.1 First order conditions

With this assumption, the new forms of equations (31) and (30) are:

$$(h_2^*)^2 + \beta h_2^* + \theta = 2h_1^* h_2^* \quad (47)$$

$$\beta(\bar{n}_1 + n_2^*) + 2n_2^*(h_2^* - h_1^*) = 0 \quad (48)$$

To which we add the hours constraint:

$$\bar{n}_1 h_1^* + n_2^* h_2^* = H \quad (49)$$

Equations (47), (48) and (49) form a non-linear system with three endogenous variables.

Linearity of the subsystem (48 and 47) for a given value of n_2^* enables however to express h_1 and h_2 as functions of the endogenous employment of type 2 workers n_2^*, yielding a feature of the solutions in comparison with average working time $\bar{h} = \dfrac{H}{\bar{n}_1 + n_2^*}$:

$$h_1^* = \bar{h} + \frac{\beta}{2} \quad (50)$$

$$h_2^* = \bar{h} - \frac{\beta}{2}\frac{\bar{n}_1}{n_2^*} \quad (51)$$

Equation (49) has distinct real roots only if its determinant $(\beta - 2h_1^*)^2 - 4\theta > 0$, implying $\beta - 2h_1^* > 2\sqrt{\theta}$ or $\beta - 2h_1^* < -2\sqrt{\theta}$. The first case is not compatible with (50) since $\bar{h} > 0$, the second case implies: $h_1^* > \sqrt{\theta} + \dfrac{\beta}{2}$ and with (50) $\bar{h} > \sqrt{\theta}$. Since notional working time for both types is $\hat{h} = \sqrt{\theta}$, first order conditions imply $\bar{h} > \hat{h}$.

4.1.2 Second order conditions

We show in Appendix 4 that second order necessary conditions for cost minimization imply:

$$\left(\frac{n_2^*}{\bar{n}_1}\right)^3 > \frac{\beta^2}{4\theta} \quad (52)$$

indicating that the non-discriminating mixed regime is only compatible with a minimum employment of type 2 workers (denoted by n_2^o), determined by parameters in inequality (52).

The model accounts for the local discontinuity of the demand for type 2 workers as a function of H.

Proposition 7 *The optimal number n_2^* of P_2 contracts offered by the employer (n_2^*) is the superior root of:*

$$\frac{H^2}{(\bar{n}_1 + n_2^*)^2} = \theta + \left(\frac{\beta}{2}\frac{\bar{n}_1}{n_2^*}\right)^2 \tag{53}$$

Proof. See Appendix A.5 for the formal proof. ■

Numerical simulations also show that the lower root corresponds to a local maximum of the cost (of no interest for us), while the superior root corresponds to the minimum of the cost function (i.e., our solution). See Figure 6 in Appendix 5.

Proposition 8 *For interior solutions, n_2^* i.e., the optimal number of P_2 contracts (demand for type 2 workers), is increasing in H.*

Proof. See Appendix A.5 for the proof. ■

This comparative static rule could appear intuitively trivial; in fact, it is not, since for an increased hours constraint, the employer also controls the two working times h_1 and h_2.

In Figure 1 we represent how n_2^* varies with H, for $n_1 = 10, \beta = 0.10$ and $\theta = 0.20$. We chose $H > 7.5$ and verify later that for this amount of hours the mixed employment regime prevails (the regime shift was presented in Section 2.3).

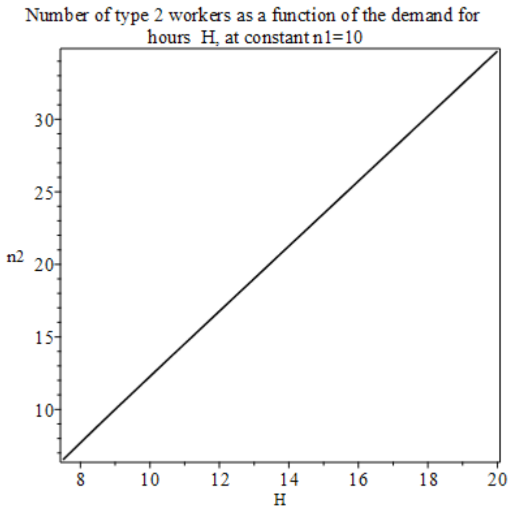

Figure 1: Demand for type 2 workers, depending on H

For the optimal n_2^*, equations (50) and (51) define the optimal working hours. Figure 2 presents how working hours vary when H increases. The red (lower) curve represents h_1^* and the blue (upper) curve h_2^*. The horizontal line corresponds to the notional working time, $\hat{h} = \sqrt{\theta} = 0.45$.

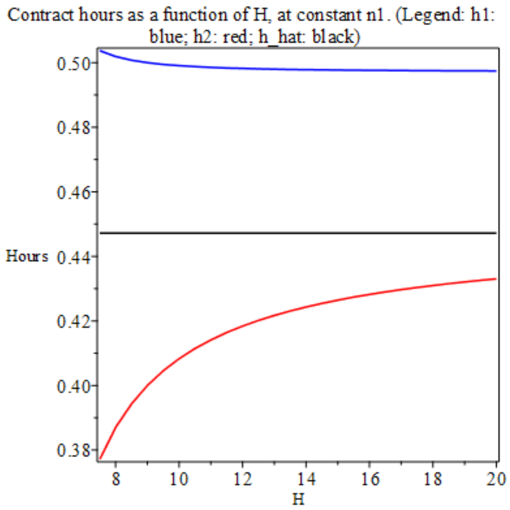

Figure 2: Contract hours by worker type, depending on H

4.2 Wage rate inequality

Once we determined the working hours specific to each contract, we can determine the wage rate for each type.

Type 2 workers get no surplus, thus $w_2 = v_2(h_2^*)/h_2^* = h_2^* + \beta$. According to equation (44), the wage of type 1 workers is: $w_1 = \dfrac{v_1(h_1^*) + \beta h_2^*}{h_1^*} = \dfrac{v_1(h_1^*)}{h_1^*} + \beta \dfrac{h_2^*}{h_1^*} = h_1^* + \beta \dfrac{h_2^*}{h_1^*}$.

We analyze the paradoxical situation where the least demanding type 1 workers (who also do long hours) will earn a higher hourly wage than the more demanding type 2 workers (who do shorter hours). It can be shown that:

Proposition 9 *The hourly wage inequality $w_1 > w_2$ holds if the average working time $\bar{h} > \dfrac{\beta}{2}$.*

Proof. $w_1 > w_2 \Leftrightarrow h_1^* + \beta \dfrac{h_2^*}{h_1^*} > h_2^* + \beta$ or $h_1^* \left(h_1^* - h_2^*\right) > \beta \left(h_1^* - h_2^*\right)$. It has been proven (Proposition 4) that $h_1^* > h_2^*$; therefore the condition for $w_1 > w_2$ is simply $h_1^* > \beta$, or, replacing h_1^* by its expression (50) by $\bar{h} > \dfrac{\beta}{2}$. ■

Notice that inequality $\bar{h} > \dfrac{\beta}{2}$ is based on the endogenous value n_2^* included in the definition of $\bar{h}$.

Proposition 10 *When the number of demanded hours increases indefinitely ($H \to \infty$), the hourly wage inequality $w_1 > w_2$ holds if $\sqrt{\theta} > \dfrac{\beta}{2}$.*

Proof. If $H \to \infty, n_2 \to \infty$ and from its definition $\bar{h} = \dfrac{H}{(\bar{n}_1 + n_2^*)} \to \dfrac{H}{n_2^*} = h_2^*$. But when

$n_2 \to \infty$, it has been shown (42) that $h_2^* \to \hat{h}_2$. The notional working time $\hat{h}_2$ is the solution of: $v'(h) = \dfrac{v(h)+\theta}{h}$; in our assumptions, $\hat{h}_2 = \sqrt{\theta}$. The limit of condition $\bar{h} > \dfrac{\beta}{2}$ is therefore: $\sqrt{\theta} > \dfrac{\beta}{2}$ ■

Figure 3 shows the evolution in hourly wages as the demand for hours H increases. Both wages increase in H, however, the less demanding type 1 workers, who also work long hours (blue, upper curve), receive a higher hourly wage than type 2 workers (red, lower curve).

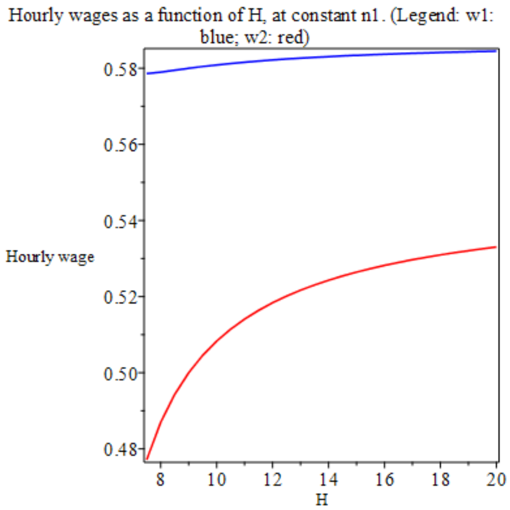

Figure 3: Contract wage rates by worker type, depending on H

4.3 Cost analysis

Finally, we would like to check that our simulations match well the case in which the employer prefers to use both type of workers rather than to use only the less demanding type 1.

In Section 2.3 we analyzed the cost functions associated to the different employment regimes, depending on the total demand for hours H, and given the bounded number of type 1 workers, $\bar{n}_1$.

In the quadratic compensation case, $v_1(h) = h^2$, the notional working time of these least demanding workers is $\hat{h}_1 = \sqrt{\theta}$. As long as $H < H_1 = \bar{n}_1\sqrt{\theta}$ (in our simulation, $H_1 = 10\sqrt{0.20} \approx 4.5$), the firm should offer only the notional contract of type 1 workers and hire only them.

For $H > H_1$, the cost of hiring only type 1 workers becomes convex. The firm *might* want to hire both type of workers, provided that the cost of doing so is lower than using only type 1. To determine which of the two employment regimes is preferred by the firm, we compare the cost of using only type 1 workers $C_1(H) = \bar{n}_1 \left[\left(\frac{H}{\bar{n}_1} \right)^2 + \theta \right]$ and the cost of labor in the mixed regime, given the optimal values as resulting from the cost minimization problem, $C^*(H)$.

$$\begin{aligned} C^*(H) &= (\bar{n}_1 + n_2^*)\theta + \bar{n}_1 h_1^* w_1^* + n_2^* h_2^* w_2^* \\ &= (\bar{n}_1 + n_2^*)\theta + \bar{n}_1 h_1^* \left(h_1^* + \beta \frac{h_2^*}{h_1^*} \right) + n_2^* h_2^* (h_2^* + \beta) \\ &= (\bar{n}_1 + n_2^*)\theta + \bar{n}_1 \left[(h_1^*)^2 + \beta h_2^* h_1^* \right] + n_2^* \left[(h_2^*)^2 + \beta h_2^* \right]. \end{aligned} \tag{54}$$

Numerical simulations show that for $H > \check{H} = 7.5$ the mixed employment regime is indeed less expensive than employing only type 1 workers.

In Figure 4 we represent these two cost functions ($C_1(H)$ in black, the upper curve; and $C^*(H)$ in green, the lower curve).

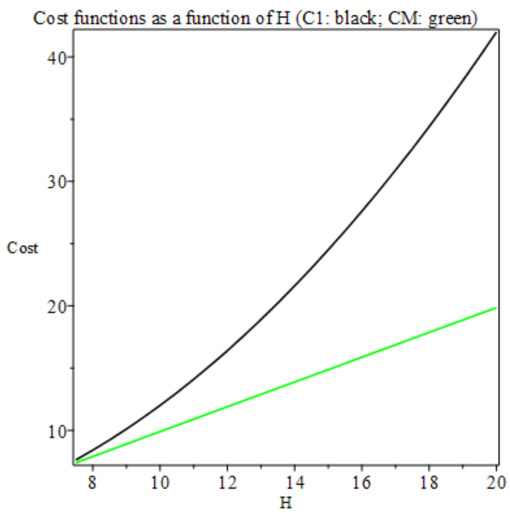

Figure 4: Cost functions

In the numerical example, the cost function (12) becomes:

$$C(H) = \begin{cases} \hat{C}_1(H) & \text{for } 0 < H \leq 4.5 \quad \text{(Type 1 only, notional hours)} \\ C_1(H) & \text{for } 4.5 < H \leq 7.5 \quad \text{(Type 1 only, longer hours)} \\ C^*(H) & \text{for } 7.5 < H \quad \text{(Non-discriminating mixed regime)} \end{cases} . \tag{55}$$

This analysis reveals the emergence of a discontinuity in the demand for type 2 workers. Indeed, for $H < 7.5$, $n_2^* = 0$; for $H = 7.5$, $n_2^* = 6.53 > 0$, and for $H > 7.5$, $n_2^* > 6.53$. This consequence of the model can be extremely problematic in periods of economic crisis, as smooth downward changes in the total demand for hours can translate into massive cuts in the employment of type 2 workers.

5 Conclusion

Confronted with a wealth of data exhibiting frequent discrepancies in labour contracts, especially consisting in compensation differences for apparently equivalent workers, many researchers have, often successfully, attempted explanations involving discriminating policies practised by employers. Our text does not contradict the relevance of the abundant resulting literature, but suggests the existence of alternative possibilities. It shows that the framework of contemporary contract theory, providing more comprehensive tools to analyze labour relations, is able to bring out situations in which technically perfectly substitutable workers, freely choosing their contracts may be given different working compensations including different hourly wage rates. In particular, our analysis shows how to formalize the external effects of preferences of one type of worker, on the contract offered to the other type.

Our analysis focuses on the non-trivial case in which the less demanding workers are in scarce supply. If contract discrimination were possible, these workers would work long hours and be paid lower hourly wages than the more demanding type worker. Our results revel the necessary conditions for that a minority of less demanding workers can be offered higher wage rates than their more exacting competitors. It enables to explain wage differentials "from nothing", their origin being an invisible scarcity constraint impossible to trace by econometrics from individual data. If one agrees that women have better work alternatives outside the labor market (Cain, 1986; OECD, 2018)[8] , and demand higher compensation than men for any working time, in our model the (hourly) wage gender gap in favour of male workers would only reveal these differences in preferences and the scarcity of male workers, and not differences in productivity, negative perceptions or women penalizing stereotypes.

We have also shown how in the non-discrimination case, contract hours depend on the structure

[8] Related to child rearing, for instance, as indicated by the OECD (2018).

of the preferences; for time increasing compensation differences, the less demanding workers will do longer hours than the more demanding workers, yet the opposite difference prevailing in the (maybe less plausible) situation of time decreasing differences.

Finally, the model allowed us to analyze how working time, hourly wages and employment respond in a somewhat unconventional way to changes in the total demand for hours. In particular, we show that the demand for type 2 workers is being exposed to discontinuities. In a macroeconomic perspective, the discontinuity in the demand for type 2 workers can explain why small fluctuations in global demand are sometimes associated to large fluctuations in some type of employment.

These results do not rely on extremely restrictive assumptions; the use of the quadratic form for the compensation function just allowed us to provide explicit solutions and numerical simulations to better guide our intuition. The cost minimization assumption is quite general, as any firm, be it a profit maximizer or not, should address it as a first stage decision. In this paper, we studied the case of a single firm, with no market power, seeking to hire hours from a pool of available workers in which the less demanding are in scarce supply. Our results would not change if we consider the case of several firms, each aiming to hire a predetermined amount of hours, in a competitive labor market. While any employer would prefer to hire only the least demanding type of worker, some firms in consequence of their scarcity, have no choice but to attract the more demanding workers, by posting a contract tailored for them. In this case, the problem that we have analyzed at the firm level would be identically transposed at the sector level.

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A Online Appendix

A.1 Independent cost minimization: uniqueness

Assumptions related with compensation function $v(h)$ are: $v'(h) > 0$ and $v''(h) > 0$ implying strict convexity. The saturated participation constraint takes the simple form $c = v(h)$. The non-wage fixed cost of a working place is θ labor and n is the number of hired workers. The objective consists in minimizing the cost of H, cost noted $C(H) = n\left[v(h) + \theta\right]$ with $H = nh$, or after substitution:

$$C(H) = \frac{H}{h}\left[v(h) + \theta\right]. \tag{56}$$

The cost minimizing working time or "notional working time" is:

$$\hat{h} = \arg\min_{h}\left\{\frac{v(h) + \theta}{h}\right\}. \tag{57}$$

Let $\hat{h} \equiv \arg\min\{\gamma(h)\}$ where $\gamma(h) = \left\{\frac{v(h)+\theta}{h}\right\}$. From definition of $\gamma(h)$, its first derivative is

$$\gamma'(h) = \frac{hv'(h) - [v(h) + \theta]}{h^2}. \tag{58}$$

First order condition $\gamma'(h) = 0$ is therefore $hv'(h) - [v(h) + \theta] = 0$, implying:

$$v'(\hat{h}) = \frac{v(\hat{h}) + \theta}{\hat{h}}. \tag{59}$$

(If $\gamma'(h) = 0$,the second order condition $\gamma''(h) > 0$ is simultaneously satisfied).

Define:

$$\varphi(h) = v(h) - hv'(h). \tag{60}$$

Condition (59) is equivalent to:

$$\hat{\varphi(h)} = -\theta. \tag{61}$$

Convexity of $v(h)$ implies $\varphi'(h) = -hv''(h) < 0$, therefore $\varphi(h)$ is monotonously decreasing in $h, \forall h > 0$. Therefore, if the solution of (59) exists, it is unique, defining a function $\hat{h}(\theta)$.

A.2 Mixed employment: an example of discriminating solution

With perfect discrimination, contracts may be determined independently, and the minimized cost of hours from type 2 is constant (4).

The cost of hours from the limited number of type 1 workers, given by the function (8) is monotonously increasing. The employer starts to hire type 2 workers when $C_1'(H) \geq \hat{C}_2'(H)$, the hours from type 2 becoming cheaper.

The marginal cost of hours from the two types are equal if:

$$C_1'(H) = v_1'\left(\frac{H}{\bar{n}_1}\right) = v_2'(\hat{h}_2) = \frac{v_2(\hat{h}_2) + \theta}{\hat{h}_2}. \tag{62}$$

Equation (62) has a unique solution indicating the relevant switching point when discrimination is feasible.

We show that when *discrimination is possible*, and if $v_1(h) = h^2$ and $v_2(h_2) = h^2 + \beta h$, cost minimization of H hours implies the wage rate ordering:

$$w_2^* = \frac{v_2(h_2^*)}{h_2^*} > w_1^* = \frac{v_1(h_1^*)}{h_2^*}. \tag{63}$$

Without incentive compatibility constraints,the relevant Lagrangian is:

$$L\left(h_1, h_2, n_{2,}\lambda\right) = \bar{n}_1\left[v_1(h_1) + \theta\right] + n_2\left[v_2(h_2) + \theta\right] - \lambda\left[\bar{n}_1 h_1 + n_2 h_2 - H\right]. \tag{64}$$

First order conditions:

$$L_1 = \bar{n}_1\left[v_1'(h_1^*) - \lambda\right] = 0 \tag{A.65}$$

$$L_2 = n_2\left[v_2'(h_2^*) - \lambda\right] = 0 \tag{A.66}$$

$$L_{n_2} = v_2(h_2^*) + \theta - \lambda h_2^* = 0 \tag{A.67}$$

$$L_\lambda = -\left(\bar{n}_1 h_1^* + n_2^* h_2 - H\right) = 0 \tag{A.68}$$

Since $v_2'(h_2^*) = \lambda$, then $L_{n_2} = 0$ implies $\dfrac{v_2(h_2^*) + \theta}{h_2^*} = v_2'(h_2^*)$ or $h_2^* = \hat{h}_2$.

From assumptions pertaining to compensation functions: $v_2(h) = h^2 + \beta h$,
$\dfrac{v_2(h_2^*) + \theta}{h_2^*} = v_2'(h_2^*) \Leftrightarrow \dfrac{(h_2^*)^2 + \beta h_2^* + \theta}{h_2^*} = 2h_2^* + \beta \Rightarrow h_2^* = \hat{h}_2 = \sqrt{\theta}.$
From $L_1 = L_2 = 0$, $v_1'(h_1^*) = v_2'(h_2^*) \Rightarrow 2h_1^* = 2h_2^* + \beta = 2\sqrt{\theta} + \beta \Rightarrow h_1^* = h_2^* + \dfrac{\beta}{2} = \sqrt{\theta} + \dfrac{\beta}{2}.$

We can determine the optimal wage rates:
$w_1^* = \dfrac{v_1(h_1^*)}{h_1^*} = \dfrac{(h_1^*)^2}{h_1^*} = h_1^* = \sqrt{\theta} + \dfrac{\beta}{2}.$
$w_2^* = \dfrac{v_2(h_2^*)}{h_2^*} = \dfrac{v_2(\hat{h}_2)}{\hat{h}_2} = \dfrac{(\hat{h}_2)^2 + \beta\hat{h}_2}{\hat{h}_2}$ where $\hat{h}_2 = \sqrt{\theta}$ implying: $w_2^* = \sqrt{\theta} + \beta.$

We verify that:

$$w_2^* = \sqrt{\theta} + \beta > w_1^* = \sqrt{\theta} + \frac{\beta}{2}. \tag{69}$$

The wage rate of the more demanding type is higher than the wage rate of the least demanding type.

A.3 Binding limitation of type 1 workers

Considering the problem:

$$\min \{n_1 [v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta] + n_2 [v_2(h_2) + \theta]\} \tag{70}$$

with $[n_1h_1 + n_2h_2 - H]$ and $n_1 \leq \bar{n}_1$.

Control variables are (n_1, n_2, h_1, h_2); the corresponding Lagrangian is:

$$L(n_1, n_2, h_1, h_2, \lambda) = n_1 [v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta] + n_2 [v_2(h_2) + \theta] - \lambda [n_1h_1 + n_2h_2 - H]. \tag{71}$$

If $0 < n_1^* < \bar{n}_1$, first order necessary conditions imply:

$$\frac{\partial L}{\partial n_1} = [v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta] - \lambda h_1 = 0 \tag{A.72}$$

$$\frac{\partial L}{\partial n_2} = v_2(h_2) + \theta - \lambda h_2 = 0 \tag{A.73}$$

$$\frac{\partial L}{\partial h_1} = n_1 v_1'(h_1) - \lambda n_1 = 0 \tag{A.74}$$

and therefore:

$$\frac{v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta}{h_1} = \frac{v_2(h_2) + \theta}{h_2} = \lambda = v_1'(h_1). \tag{75}$$

This equality indicates that if the availability constraint is not binding, the cost of hours obtained from each type should coincide.

But (75) implies:

$$\frac{v_1(h_1) - v_1(h_2)}{h_1 - h_2} = \lambda = v_1'(h_1). \tag{76}$$

Convexity of $v_1(h)$ implies $v_1(h_2) - v_1(h_1) > (h_2 - h_1)v_1'(h_1) \Rightarrow v_1(h_1) - v_1(h_2) < (h_1 - h_2)v_1'(h_1)$ or $\dfrac{v_1(h_1) - v_1(h_2)}{h_1 - h_2} < v_1'(h_1)$, contradicting (76).

A.4 Second order conditions for cost minimization

We consider the case of quadratic compensation functions and use the constraint to reduce the problem to a form of free variables minimization. Since for $n_1 = \bar{n}_1$, n_2 is explicitly determined by the hours constraint and the choice of h_1 and h_2, the objective function to be minimized is:

$$m(h_1, h_2) = \bar{n}_1 \left[v_1(h_1) + v_2(h_2) - v_1(h_2) + \theta\right] + \frac{H - \bar{n}_1 h_1}{h_2} \left[v_2(h_2) + \theta\right]. \tag{77}$$

First order derivatives are:

$$m_1(h_1, h_2) = \bar{n}_1 v_1'(h_1) - \frac{\bar{n}_1}{h_2} \left[v_2(h_2) + \theta\right] \tag{A.78}$$

$$m_2(h_1, h_2) = \bar{n}_1 \left[v_2'(h_2) - v_1'(h_2)\right] - \frac{H - \bar{n}_1 h_1}{h_2^2} \left[v_2(h_2) + \theta\right] + \frac{H - \bar{n}_1 h_1}{h_2} v_2'(h_2). \tag{A.79}$$

After substitutions, since $v_1(h) = h^2$, $v_2(h) = h^2 + \beta h$ and $\dfrac{H - \bar{n}_1 h_1}{h_2} = n_2$.

First order conditions are:

$$m_1(h_1, h_2) = 2\bar{n}_1 h_1 - \bar{n}_1 h_2 - \beta \bar{n}_1 - \bar{n}_1 \frac{\theta}{h_2} = 0 \tag{A.80}$$

$$m_2(h_1, h_2) = \bar{n}_1 \beta + n_2 h_2 - n_2 \frac{\theta}{h_2} = 0. \tag{A.81}$$

From (80) and (81), the elements of the Hessian matrix $\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ are:

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} 2\bar{n}_1 & \bar{n}_1 \left(\dfrac{\theta}{h_2^2} - 1\right) \\ \bar{n}_1 \left(\dfrac{\theta}{h_2^2} - 1\right) & \dfrac{2n_2}{h_2^2}\theta \end{pmatrix}. \tag{82}$$

Second order necessary conditions for minimization are: $m_{11} > 0$ and $m_{22} > 0$ (always fulfilled), and $\det \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} > 0$ which is fulfilled iff:

$$\frac{4\bar{n}_1 n_2}{h_2^2}\theta > \bar{n}_1^2 \left(\frac{\theta}{h_2^2} - 1\right)^2 \Leftrightarrow 4\theta n_2 > \bar{n}_1 \left(\frac{\theta}{h_2} - h_2\right)^2. \tag{83}$$

From (80) $\dfrac{\theta}{h_2} = 2h_1 - h_2 - \beta$ and $\dfrac{\theta}{h_2} - h_2 = 2(h_1 - h_2) - \beta$ and from (80) and (81) $2(h_1 - h_2) - \beta = \dfrac{\bar{n}_1}{n_2}\beta$.

Thus $\det \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} > 0$ iff $4\theta n_2 > \bar{n}_1 \left(\dfrac{\bar{n}_1}{n_2}\beta\right)^2$.

This condition amounts to a minimum relative participation of type 2, as determined by:

$$\left(\frac{n_2}{\bar{n}_1}\right)^3 > \frac{\beta^2}{4\theta}, \qquad (84)$$

and a minimum amount of total hours.

A.5 The demand for type 2 workers

The computed values of working times for a given (endogenous) number of type 2 workers:

$$h_1^* = \bar{h} + \frac{\beta}{2} \qquad (A.85)$$

$$h_2^* = \bar{h} - \frac{\beta}{2}\frac{\bar{n}_1}{n_2} \qquad (A.86)$$

where $\bar{h} = \frac{H}{(\bar{n}_1 + n_2)}$.

Introducing these expressions in equation (49): $h_2^2 + \beta h_2 + \theta = 2h_2 h_1$ we obtain:

$$\underbrace{\frac{H^2}{(\bar{n}_1 + n_2)^2}}_{l(n_2)} = \underbrace{\theta + \left(\frac{\beta\bar{n}_1}{2n_2}\right)^2}_{r(n_2)} \qquad (87)$$

or, equivalently,

$$\underbrace{\frac{n_2^2 H^2}{(\bar{n}_1 + n_2)^2}}_{L(n_2)} = \underbrace{\theta n_2^2 + \left(\frac{\beta\bar{n}_1}{2}\right)^2}_{R(n_2)} \qquad (88)$$

The demand for type 2 workers is the implicit solution to the former equation.

Derivatives with respect n_2 are:

$$L'(n_2) = \frac{2\bar{n}_1 n_2}{(\bar{n}_1 + n_2)^3} H^2 \qquad (A.89)$$

$$R'(n_2) = 2\theta n_2 \qquad (A.90)$$

From (graphic) analysis of $L(n_2)$ and $R(n_2)$, the root of (88) *is real and unique* if $L(n_2) = R(n_2)$ and $L'(n_2) = R'(n_2)$.

$L'(n_2) = R'(n_2) \Rightarrow H^2 = \theta\frac{(\bar{n}_1 + n_2)^3}{\bar{n}_1}$ and with (87), the condition implies: $\frac{\bar{n}_1 + n_2}{\bar{n}_1}\theta = \theta + \left(\frac{\beta\bar{n}_1}{2n_2}\right)^2$ and finally:

$$\left(\frac{n_2}{\bar{n}_1}\right)^3 = \frac{4\theta}{\beta^2}. \qquad (91)$$

The possible unique positive root, denoted by n_2^0, is defined by (91) $\left(\frac{n_2^0}{\bar{n}_1}\right)^3 = \frac{4\theta}{\beta^2}$ and corresponding to the switching value of $\check{H}$.

For $H > \check{H}$ there are two real positive roots for equation (88), but the smaller one is smaller than n_2^0 implying $\left(\frac{n_2}{\bar{n}_1}\right)^3 < \frac{4\theta}{\beta^2}$ and thus contradicting the second order condition for cost minimization (84).

Application: $\frac{dn_2^*}{dH} > 0$

From first order conditions, equation (88) (in the main text) obtains.

In the neighborhood of an interior solution, n_2^* is a continuous and differentiable function of H, noted $g(H)$ and (88) is written:

$$H^2 \left[\bar{n}_1 + g(H)\right]^{-2} = \theta + \left(\frac{\beta\bar{n}_1}{2}\right)^2 g^{-2}(H) \tag{92}$$

After derivating both members with respect to H :

$$g'(H) = \frac{H\left[\bar{n}_1 + g(H)\right]}{H^2\left[\bar{n}_1 + g(H)\right]^{-3} - \left(\frac{\beta\bar{n}_1}{2}\right)^2 g^{-3}(H)}. \tag{93}$$

The condition for $\frac{dn_2^*}{dH} = g'(H) > 0$ is:

$$H^2 \left[\bar{n}_1 + g(H)\right]^{-3} > \left(\frac{\beta\bar{n}_1}{2}\right)^2 g^{-3}(H) \tag{94}$$

or equivalently:

$H^2 \left[\bar{n}_1 + g(H)\right]^{-2} > \left\{\left(\frac{\beta\bar{n}_1}{2}\right)^2 g^{-3}(H)\right\}\left[\bar{n}_1 + g(H)\right]$; and from (92):

$\theta + \left(\frac{\beta\bar{n}_1}{2}\right)^2 g^{-2}(H) > \bar{n}_1\left(\frac{\beta\bar{n}_1}{2}\right)^2 g^{-3}(H) + \left(\frac{\beta\bar{n}_1}{2}\right)^2 g^{-2}(H)$

Therefore: $\frac{dn_2^*}{dH} > 0$ iff : $\theta > \bar{n}_1\left(\frac{\beta\bar{n}_1}{2}\right)^2 g^{-3}(H)$ i.e. iff $\theta > \frac{\beta^2}{4}\left(\frac{\bar{n}_1}{n_2^*}\right)^3$.

From second order necessary condition (84), this condition is necessarily fulfilled.

Numerical simulations

Figure 5 represents the functions $l(n_2)$ and $r(n_2)$ for $H = 7.5$, which is the downward bound for existence of the mixed employment regime. The other parameters are similar to those used in the main text numerical simulation ($\beta = 0.10$, $\theta = 0.20$, $\bar{n}_1 = 10$). As we can see, in this case, equation (88) has two positive roots, $n_2^l = 0.96$ and $n_2^h = 6.53$.

We study the cost of producing H hours for varying n_2 (including the two roots). We recall that in the mixed employment regime, the cost of producing a predetermined amount of hours

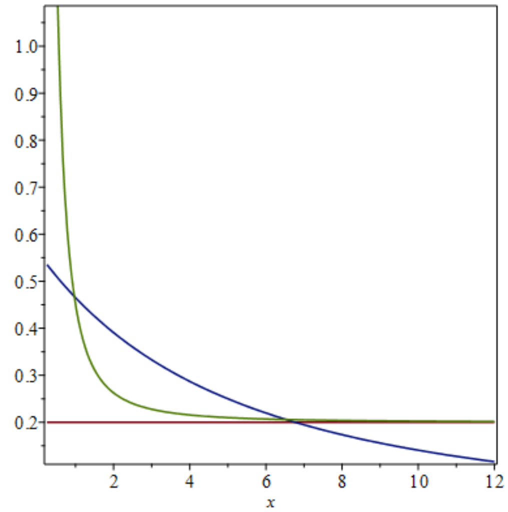

Figure 5: The two positive solutions to Equation 88

$(H = 7.5)$ can be written as a function of n_2 only (see equation 54), all other variables being set at their optimal level (depending on n_2):

$$C^*(H = 7.5, n_2) = (\bar{n}_1 + n_2)\theta + \bar{n}_1 \left[(h_1^*)^2 + \beta h_2^* h_1^* \right] + n_2^* \left[(h_2^*)^2 + \beta h_2^* \right] \tag{95}$$

where $h_1^* = h_1^*(n_2)$ and $h_2^* = h_2^*(n_2)$. Figure 6 represents the cost of producing $H = 7.5$ depending on n_2:As revealed by the analysis of the second order conditions, the graph corroborates that

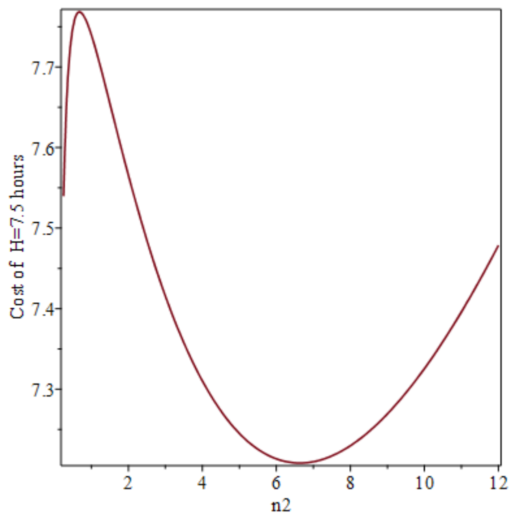

Figure 6: Cost as a function of n_2, for optimal hours and wage rates

that the larger root $(n_2^h = 6.53)$ is the solution to the cost minimization problem, the lower one corresponding to a local maximum.

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