Optimal Ex Post Risk Adjustment in Markets with Adverse Selection *

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Abstract

This paper studies general health insurance markets. It proposes an ex post risk adjustment scheme that discourages risk selection and promotes efficient competition. Under the proposed risk adjustment scheme, the regulator engages in transfers that are conditional on the ex post profits of insurers. The risk adjustment scheme is entirely budget balanced, as it does not call for government subsidies, and requires the regulator to hold minimal information to implement it. Equilibrium is shown to exist and be efficient in any environment with a finite number of types and states even if single-crossing is not satisfied.

KEYWORDS: Health insurance, risk selection, risk adjustment, efficiency

JEL CLASSIFICATION: D82, D86, I10, I13, I18

1 INTRODUCTION

In many countries with voluntary, private health insurance markets, open enrolment and community rating act as remedies against direct price discrimination that excludes high-

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risk individuals from the market.[1,2] However, community rating can lead to death spirals, according to which high-risk individuals opt for insurance plans with low premiums, raising the cost of these plans and eventually causing a rise in premiums and a potential crowding out of low-risk individuals.[3] Insurers attempt to circumvent death spirals by targeting the relatively healthy and low-risk individuals via indirect price discrimination. This practice, known as *risk selection*, can eventually make health insurance unaffordable to high-risk individuals and, hence, constitutes a major concern for policy makers.[4] In their pioneering contribution, Rothschild and Stiglitz (1976) demonstrate why and how risk selection is profitable in insurance markets with adverse selection and the potential "destructive competition" that this might entail.

Risk adjustment attempts to correct risk selection by rewarding (punishing) insurers for enrolling high-risk (low-risk) individuals.[5] Some sort of risk adjustment appears in most private health insurance markets.[6] Two main risk adjustment schemes are implemented in

[1]In health insurance markets, community rating requires insurers to offer policies within a given territory at the same price to all individuals without medical underwriting, regardless of their health status. See Pauly (1970) for an early study on the effects of community rating in insurance markets. Open enrolment requires insurers to accept every application. Usually, an annual open enrolment period is determined during which every individual can opt for health insurance at community rated premiums. Insurers can decline applicants outside the annual open enrolment period to protect themselves from individuals who choose to opt for health insurance after they are diagnosed with a medical condition that requires health care.

[2]The effects of open enrolment and community rating in health insurance markets depend on whether participation in the market is voluntary or mandatory. In mandatory markets with no open enrolment, insurers might deny coverage to high-cost individuals. Without community rating, high-risk individuals and individuals with preexisting conditions have to pay a considerable premium. In that case, community rating makes insurance "fairer".

[3]See, for instance, Buchmueller and DiNardo (2002) and Sasso and Lurie (2009).

[4]Early empirical literature finds evidence of risk selection in insurance markets (e.g., Riley, Rabey, and Kasper (1989); Riley, Lubitz, and Rabey (1991) Hill and Brown (1990, 1992), Lichtenstein et al. (1991), Strumwasser et al. (1989), van Barneveld et al. (2000), etc.). Shen and Ellis (2002) examine the potential profitability of risk selection using different risk-adjustment models. Their simulations reveal that profits from risk selection remain substantial. The Affordable Care Act (ACA) placed substantial weight on risk selection and promoted policies to alleviate it. Recently, economists have shown renewed interest in the identification of risk selection in the US health insurance market. For instance, Kuziemko, Meckel, and Rossin-Slater (2013) find evidence that insurers select low-cost individuals in Medicaid managed care in Texas. Brown et al. (2014) show that insurers risk select in Medicare Advantage, whereas Aizawa and Kim (2015) show that mass and direct mail advertising targets low-cost customers in Medicare Advantage.

[5]Van De Ven and Ellis (2000) define risk adjustment as "...the use of information to calculate the expected health expenditures of individual consumers over a fixed interval of time (e.g., a month, quarter, or year) and set subsidies to consumers or health plans to improve efficiency and equity...".

[6]Van De Ven and Ellis (2000) document that risk adjustment is implemented in seventeen out of the eighteen markets they consider.

practice: *ex ante* (or prospective) and *ex post* (or retrospective). In an ex ante risk adjustment scheme, every individual is evaluated (in terms of riskiness) based on observable characteristics (e.g., age, gender, pre-existing conditions, chronic diseases), and the regulator subsidises insurers for every individual they enrol. In an ex post risk adjustment scheme, the regulator subsidises insurers based on the (ex post) realised cost of health care.[7] This paper proposes an alternative ex post risk adjustment scheme and shows how this discourages risk selection and promotes efficiency in general private health insurance markets.

The analysis is performed in a market inhabited by a continuum of risk-averse consumers. Each consumer can be in a health state that requires costly health care. There is a finite set of possible consumer types that differ in their risk of requiring health care, their wealth and the cost of receiving care. Insurers compete in insurance plans but are unable to directly price discriminate among consumers due to unobservable characteristics correlated with risk and/or community rating.

The proposed risk adjustment scheme engages in transfers between the regulator and every insurer based on the realised profits of all insurers. The rationale behind such a risk adjustment scheme results from the observation that when an insurer engages in risk selection, it imposes a negative externality on its rivals because the rivals bear the burden of serving the high-risk individuals. Therefore, transfers force insurers to internalise this negative externality. The risk adjustment scheme works as follows. If an insurer realises strictly positive profits while at least one of the remaining insurers realises strictly negative profits, the profit-making insurer pays a premium that is collected by the regulator and evenly distributed to the loss-making insurers as a subsidy. Under this scheme, equilibrium exists for all possible parameters, and every equilibrium allocation is efficient.[8] This result has the following intuitive explanation. Suppose that efficiency requires cross-subsidisation.[9,10] As-

[7]Ex post risk adjustment is sometimes called *claims equalisation* because the regulator engages in transfers to "equalise" the ex post claims.

[8]Perhaps as expected, when the transfers are set equal to zero, a pure strategy equilibrium does not exist so long as the well-known least-costly separating allocation is not efficient. This result is formally proven in Dosis (2017).

[9]This is the range of parameters in which Rothschild and Stiglitz (1976) show that entrants can realise strictly positive profits by engaging in risk selection. The notion of efficiency is constrained Pareto efficiency, i.e., Pareto efficiency subject to incentive constraints.

[10]Cross-subsidisation occurs when some insurance policies realise strictly negative profits while others re-

sume that all insurers offer an efficient (i.e., zero-profit) menu of contracts. Would an insurer engage in risk selection? By doing so, it harms its rivals (i.e., the fundamental negative externality mentioned above). If the transfer is set equal to the profits earned by that insurer, then any gains from risk selection will be outweighed by the transfer. This renders the deviation unprofitable.

A fine detail in the argument described above is that any risk adjustment scheme should not distort efficient competition. In other words, a regulator would like to assure that companies compete towards the efficient (constrained Pareto) frontier by eliminating any strictly positive profits. The risk adjustment scheme proposed in this paper appears to fulfil this requirement because for every inefficient menu of contracts, another menu exists that attracts all types (i.e., not only the most profitable ones) earns positive profits and does not entail any losses for a rival. Hence, no transfers are imposed, which means that insurers always have profitable deviations over inefficient allocations. Therefore, the overall approach is compatible with the theory of efficient competitive markets.

In addition to its simplicity, the proposed risk adjustment scheme has another notable advantage because, to be implemented, it only requires the regulator to possess information about the profits (or losses) realised by the insurers and not per se the menus of contracts that the insurers offer.[11] One can unquestionably argue that the realised profits are public information, especially in large markets with insurance corporations. Moreover, the risk adjustment scheme merely requires an insurer to make a transfer to a rival only if the latter realises strictly negative profits and the former strictly positive profits. This avoids situations in which an insurer might offer an unreasonably generous (i.e., loss-making) menu of contracts with the aim of driving its rivals out of the market. Such a menu would most likely attract the whole market, leaving the rival making zero profits and hence paying no transfer.

□ **Related Literature.** Glazer and McGuire (2000) is the first study that documents the fallacy of conventional (statistical) risk adjustment. It also proposes a scheme in a stylised

alise strictly positive profits. In that sense, a subset of policies cross-subsidises another subset.

[11] Although the transfers depend on the profits, the two insurers know how their menus determine profits, and hence, for them, the transfers depend on the menus.

model of managed care that adjusts for ex ante signals correlated with true risk. Because signals are never perfect, risk adjustment makes risk selection unprofitable. Shen and Ellis (2002) and Jack (2006) propose related models of optimal (ex ante) risk adjustment. Barros (2003) proposes an ex post risk adjustment scheme according to which insurers participate in an ex post fund and contribute based on the differences of their risk portfolios *vis-à-vis* the population's average risk portfolio. Under his scheme, risk selection motives are eliminated and insurers have incentives for cost reduction. Kifmann and Lorenz (2011) also study a model of ex post cost reimbursement and derive an optimal cost reimbursement formula. There are at least three differences between the present paper and those mentioned above. First, this paper studies general insurance environments in which consumers differ along more than one dimension. Second, this paper studies a risk adjustment scheme that, to the best of my knowledge, has not been studied previously in a theoretical model. Finally, as discussed in the introduction, the risk adjustment scheme proposed in this paper requires a regulator to hold minimal information to implement it. In fact, all that is required for the regulator is to observe the profits of the insurers, which is a fairly mild assumption.

In addition to the literature on risk selection and risk adjustment, this paper contributes to the literature on optimal interventions in insurance markets with selection. Early contributions such as Pauly (1974) and Wilson (1977) show that the government can improve welfare in insurance markets with adverse selection by applying a simple tax-subsidy scheme. Dahlby (1981) shows that compulsory insurance supplemented with private insurance leads to a Pareto improvement. Neudeck and Podczeck (1996) discuss alternative government interventions. One difference between the interventions proposed by these early papers and this paper regards the information required by the regulator to implement the policies.

This paper is also related to the literature that studies the existence and efficiency of equilibrium in competitive insurance markets with adverse selection following the early seminal contribution of Rothschild and Stiglitz (1976). Wilson (1977), Miyazaki (1977), Spence (1978) (henceforth WMS) and Riley (1979) examine alternative definitions of competitive equilibrium in screening markets.[12] Hellwig (1987) and Engers and Fernandez (1987) study

[12] Wilson (1977) and Riley (1979) motivate these restrictions as the reduced form of a dynamic process in which insurers react to the actions of rivals. Nonetheless, this interpretation bears a significant caveat because,

games that provide game-theoretic foundations for these alternative definitions of equilibrium but find that the qualitative features of the equilibrium set are different from those envisioned in WMS and Riley. More recent contributions by Asheim and Nilssen (1996), Netzer and Scheuer (2014) Diasakos and Koufopoulos (2018), Mimra and Wambach (2018) and lay game-theoretic foundations for the WMS equilibrium but in a stylised insurance market with only two possible types. Picard (2014, 2018) provides foundations for the MWS allocation in Wilson (1977) economies with more than two types. Dosis (2018) shows that a game that combines signalling and screening features restores existence and assures efficiency in environments with any finite number of types. This paper differs from this strand of the literature, as it focuses on government interventions that are implemented in practice instead of laying game theoretic foundations for efficient competition in markets with adverse selection.

Finally, the literature on Walrasian markets with adverse selection is also related to the present work. This literature begins with the seminal contribution of Prescott and Townsend (1984). Gale (1992, 1996, 2001) and Dubey and Geanakoplos (2002) modify the definition of Walrasian equilibrium to allow for asymmetric information of the form discussed in this article. Key is the determination of beliefs for the quality of all possible products. All articles prove the existence of an equilibrium. Citanna and Siconolfi (2016) and Azevedo and Gottlieb (2017) extend these results to more general environments. Bisin and Gottardi (2006) identify a fundamental negative externality that different types impose on one another, as opposed to the negative externality that insurers impose on one another as identified in this article. To solve this externality, they introduce markets for property rights. Property rights allow the implementation of efficient allocations because they force the different types to internalise this negative externality.

The remainder of the article is organised as follows. In Section 2, I describe the risk adjustment scheme in Australia that is particularly similar to that proposed in this paper.

unless an extensive-form game is specified that supports the reduced form, any such restriction seems rather *ad hoc*. As Riley states, "...[a]ny attempt to describe simple decision rules that result in an equilibrium in which expectations based on signals are fulfilled, no matter how plausible, is to some degree ad hoc." To the best of my knowledge, neither WMS nor Riley's market dynamics have solid game-theoretic foundations.

In Section 3, I describe the model and the proposed risk adjustment scheme. In Section 4, I prove the existence and efficiency of equilibrium. In Section 5, I discuss the results and the proposed risk adjustment scheme. In Section 6, I conclude the paper.

2 Risk Adjustment in Practice: The Case of Australia

A risk adjustment scheme that closely resembles that proposed in this paper appears in the Australian health insurance market.[13] Australia has a mixed public/private health insurance system. Since 1984, the public (tax-funded) insurance, called *Medicare*, provides every citizen of Australia public hospital treatment, subsidies for fee-for-service payments and drugs. Private health insurance is voluntary and provided by private insurers for services that are either already covered by Medicare (duplicate) or not (supplementary). Although the introduction of Medicare in 1984 led to a death spiral and consequently a significant decline in the share of the population that held private insurance (see Butler (2002)), government policies, in particular tax exemptions for purchasing or tax penalties for not purchasing private health insurance and lifetime community rating, made private insurance more affordable and led to an increase in the share of private health insurance holders. Currently, approximately 45% of the population has some private insurance coverage.

The private insurance market is heavily regulated. The *National Health Act* of 1953 introduced community rating and open enrolment regulations to restrict risk selection, and in 1958, a law encouraged insurers to create special accounts for those with preexisting or chronic medical conditions. At the end of each year, the government would finance any deficits in these accounts. These special accounts were the predecessors of the subsequent risk adjustment schemes. risk adjustment was introduced in 1976 alongside a cap in subsidisation of special accounts by the government. In 1989, the government withdrew all subsidies to special accounts and the scheme became purely redistributive.

The most recent reform was the *Private Health Insurance Act* of 2007. At present, consumers pay a community-rated premium to a voluntarily chosen insurer. Insurers pay and receive subsidies from a central, state-level, "solidarity fund" called the *Private Health Insur-*

[13]For further details, see Connelly et al. (2010) and Armstrong et al. (2010).

ance Risk Equalisation Trust Fund. Flows of payments are solely based on ex post claims of insurers rather than ex ante risk adjusters as is usually the case in most countries.

3 The Model

■ **The Insurance Market.** There is a measure one of consumers and two dates (today and tomorrow). Each consumer belongs to one of a finite set of types Θ, with representative element θ. The share of type-θ consumers in the population is λ^θ, with $\sum_\theta \lambda^\theta = 1$. Every consumer will be in a health state (henceforth, for simplicity, "state") tomorrow that might require health care. The set of possible health states is Ω (finite), with a representative element ω. A type-θ consumer starts with wealth W^θ. The cost of care in state ω for type θ is ℓ^θ_ω, and the probability of being in state ω, π^θ_ω, where $\sum_\omega \pi^\theta_\omega = 1$ for every θ.

To ameliorate the potential uncertainty, consumers can buy health insurance today. A health insurance plan (or health insurance policy, or health insurance contract) is denoted by $c = (p, (d_\omega)_\omega) \in \mathcal{C} = \mathbb{R}_+^{\#\Omega+1}$.[14] The terms p and d_ω specify the insurance premium and the deductible that the consumer will pay in state ω, respectively. A consumer of type θ has preferences for contracts represented by an expected utility function:

$$U^\theta(c) = \sum_\omega \pi^\theta_\omega u_\omega(W^\theta - d_\omega - p)$$

where u_ω is continuous, strictly increasing and strictly concave for every $\omega \in \Omega$. The status quo utility of type θ is $\underline{U}^\theta = \sum_\omega \pi^\theta_\omega u_\omega(W^\theta - \ell^\theta_\omega - T(W^\theta))$, where $T(W)$ is a tax penalty imposed on an uninsured individual with wealth W.[15] For the remainder of this paper, $T(W)$ is considered exogenous. For convenience, I call contract $c^\theta_o = (0, (\ell^\theta_\omega)_\omega)$ the *null contract* for type θ.

Insurers offer menus of insurance contracts (to be specified below) in the market. The set of insurers is denoted by I, with a representative element i. If type θ buys contract

[14] For an abstract finite set S, I denote by $\#S$ the cardinality of this set.

[15] In many countries with voluntary health insurance, an individual mandate policy is implemented to incentivise citizens to acquire health insurance. For instance in the US, the ACA imposed an individual mandate at the federal level. Individual mandates are intended to improve the pool of insured individuals, as healthy and low-risk individuals tend to opt out the market first. In countries with compulsory insurance, $T(W)$ is set large enough that it is extremely costly not to have health insurance.

$c = (p, (d_\omega)_\omega)$ from insurer i, then the latter earns an expected profit equal to

$$\zeta^\theta(c) = p + \sum_\omega \pi_\omega^\theta (d_\omega - \ell_\omega^\theta)$$

This profit does not depend on the identity of the insurer, i.e., insurers are symmetric pools of risk. This implies that when two insurers offer a contract with the same terms, consumers are indifferent in their ranking of these two contracts. Moreover, insurers are unable to directly price discriminate either due to a universal (pure) community-rating policy or due to asymmetric information between them and the insured individuals (or both).

A regulator is an overseer of the health insurance market, is benevolent and has full commitment power. The objective of the regulator is to maximise welfare.

□ **Remarks.** It is perhaps important at this point to highlight the similarities and differences of the model *vis-à-vis* related models of health insurance markets. In the early studies of insurance markets with imperfect information (e.g., Pauly (1974), Rothschild and Stiglitz (1976), Wilson (1977), etc.), consumer types differ along a single dimension: the probability of suffering a costly accident. Consumers are homogenous in other dimensions such as wealth, the cost of care, and risk aversion. It is evident that this model is a special case, albeit restrictive, of the model I study in this paper. In particular, in the model I study, consumers can be wealth-heterogenous and the cost of care might not be the same for all types. This allows, for instance, for geographical discrimination, as in different areas the cost of care might differ substantially. Moreover, I relax the assumption of state-independent utility that seems particularly strong when one studies models of health insurance provision, as individuals might differ in their valuation of wealth/consumption in different health states.

□ **Allocations.** Because the subject of the paper is the study of an efficient policy, it is essential to formally define efficiency. Given that the insurers are symmetric and act as pools of risk, it is conventional in these models to define efficiency through allocations. An *allocation* is a vector of contracts indexed by the set of types. Let an allocation be denoted by $\boldsymbol{c} = (c^\theta)_\theta$. Allocation $\boldsymbol{c}$ is *incentive compatible* if and only if $U^\theta(c^\theta) \geq U^\theta(c^{\theta'})$ for every $\theta, \theta' \in \Theta$. An incentive compatible allocation $\boldsymbol{c}$ is *individually rational* if and only if $U^\theta(c) \geq \underline{U}^\theta$ for every

θ. Let $Z(\boldsymbol{c}) = \sum_\theta \lambda^\theta \zeta^\theta(c^\theta)$ denote the aggregate profit of allocation $\boldsymbol{c}$. The aggregate profit of an allocation is the difference between the weighted sum of its premiums and deductibles and its expected costs. An *efficient allocation* is formally defined below:

Definition 1. *Allocation $\boldsymbol{c}$ is efficient if and only if (i) it is incentive compatible and individually rational, (ii) it yields positive profit, i.e., $Z(\boldsymbol{c}) \geq 0$, and (iii) there exists no other allocation $\hat{\mathbf{c}}$ that satisfies (i), (ii) and, $U^\theta(\hat{c}^\theta) \geq U^\theta(c^\theta)$ for every θ with the inequality being strict for at least one θ.*

Some remarks regarding the set of efficient allocations appear useful at this stage. First, efficiency, as defined here, is standard constrained Pareto efficiency subject to incentive constraints. However, I restrict attention to a subset of the set of constrained Pareto efficient allocations by imposing individual rationality constraints, one for each type. Because the market is voluntary, consumers may decide to remain uninsured and, hence, as shown below, no equilibrium can be sustained in which any of the types has payoff lower than this from her status quo. Second, the set of efficient allocations is potentially large. For instance, in the simple two-by-two insurance market studied in Rothschild and Stiglitz (1976), in which there are two possible states (accident and no accident) and two possible types (high risk and low risk) the set of efficient allocations, as characterised in Crocker and Snow (1985), consists of a continuum of allocations. Some of these involve cross-subsidies and hence are vulnerable to risk selection.

4 Risk Adjustment and Equilibrium

To fix notation, let m_i denote a menu for insurer i and $\boldsymbol{m} = (m_i)_i$ a profile of menus. Let also $\boldsymbol{m}_{-i}$ denote a profile of menus for all insurers other than i. The set of possible menus for each insurer is $\mathcal{C}^N$, where $N \geq \#\Theta$.[16] For convenience, let $m_o = (c_o^\theta)_\theta$ denote the menu that contains only the null contracts.

A *risk adjustment scheme* defines transfers from and to the regulator based on a set of ex post realised variables that might reveal risk selection. For instance, risk adjustment may

[16]The number of contracts in a menu exceeds the number of possible types. Therefore, potentially, an insurer can serve the whole market. One could allow for an infinite number of contracts, although this only adds technical difficulty without affecting the results.

condition transfers on the realised costs of insured firms. In this paper, I examine an alternative risk adjustment scheme that conditions transfers on the realised profits of insurers. Although the risk adjustment scheme depends on the realised profits, the insurers rationally expect how the profits are determined by the menus of contracts that are available in the market given the demand functions (to be specified below). Hence, without loss of generality and for notational simplicity I assume that the risk adjustment scheme depends on the menus of contracts that are available in the market. Therefore, let $t_i(\boldsymbol{m})$, denote the *net transfer* of insurer i when the profile of menus is $\boldsymbol{m}$. This can be either positive or negative. The risk adjustment scheme is denoted by $\boldsymbol{t}(\boldsymbol{m}) = \big(t_i(\boldsymbol{m})\big)_i$.

The timing of the game is described below:

Stage 1: The regulator sets the risk adjustment scheme.

Stage 2: Insurers observe the risk adjustment scheme, and each offers a menu of contracts.

Stage 3: Consumers each select at most one insurance contract or remain uninsured.[17]

In Stage 3, consumers each select a contract from all the available contracts in the market. Because the question of interest regards the study of competition and risk selection, I suppress Stage 3 through the description of a (rational) demand system. In particular, let $q_i^\theta(c|\boldsymbol{m})$ denote the demand function for contract $c \in m_i$, by type-θ consumers for insurer i, when the profile of menus is $\boldsymbol{m}$. Let $\boldsymbol{q}(\boldsymbol{m})$ denote the demand profile. For notational convenience, let $\Gamma(\boldsymbol{m}) = (\cup_i m_i) \cup m_o$ denote the union of the set of contracts available in the market. This implicitly presumes that a consumer always has access to the null contract and hence to her outside option. Consumers optimally select a contract from the set of available contracts. Therefore, the demand profile $\boldsymbol{q}(\boldsymbol{m})$ satisfies a set of rationality constraints. First, for every θ, $\boldsymbol{m}$ and i,

$$q_i^\theta(c|\boldsymbol{m}) = 0 \text{ if } \; U^\theta(c) < \max_{c' \in \Gamma(\boldsymbol{m})} U^\theta(c') \tag{1}$$

In other words, the demand for a contract is zero when this contract does not belong to the

[17] Note that, as is standard in the literature, an exclusivity assumption is imposed. Hence, no *common agency* issues are studied.

set of contracts that maximise the utility of type θ amongst the contracts that are available in the market, including the null contract, i.e., $\Gamma(\boldsymbol{m})$.

Second, for every θ, $\boldsymbol{m}$ and i and i'

$$q_i^\theta(c|\boldsymbol{m}) = q_{i'}^\theta(c'|\boldsymbol{m}) \text{ if } \; U^\theta(c) = U^\theta(c') \tag{2}$$

In other words, if firms offer contracts for which type-θ consumers are indifferent, then each firm gets an equal share of the total demand.

Third, for every θ, $\boldsymbol{m}$ and i

$$q_0^\theta(\boldsymbol{m}) + \sum_i \sum_{c\in m_i} q_i^\theta(c|\boldsymbol{m}) = \lambda^\theta \tag{3}$$

In other words, the demands sum up to the ex ante share of type θ. The term $q_0^\theta(\boldsymbol{m})$ represents the share of type-θ consumers who do not buy any insurance.

Last, for every θ, $\boldsymbol{m}$ and j

$$q_0^\theta(\boldsymbol{m}) = 0 \;\; \text{if} \;\; \underline{U}^\theta \leq \max_{c'\in\cup_i m_i} U^\theta(c') \tag{4}$$

In other words, states that this is equal to zero if some contract offered by at least one of the insurers provides a payoff weakly higher than the null contract.

Under risk adjustment scheme $\boldsymbol{t}(\boldsymbol{m})$, one can specify the expected profit of insurer i as

$$\Pi_i(\boldsymbol{m}, \boldsymbol{t}(\boldsymbol{m})) = \Phi_i(\boldsymbol{m}) + t_i(\boldsymbol{m}) \tag{5}$$

where $\Phi_i(\boldsymbol{m}) = \sum_\theta \sum_{c\in m_i} q_i^\theta(c|\boldsymbol{m})\zeta^\theta(c)$. In other words, the total profit of insurer i when offering a menu of contracts is equal to the net profits (losses) made by selling these contracts plus the (net) transfer it pays to the regulator.

A formal definition of a (subgame perfect) Nash equilibrium in pure strategies of the game played by the insurers follows:

Definition 2. *For a given risk adjustment scheme $\boldsymbol{t}(\boldsymbol{m})$, a Nash equilibrium in pure strategies consists of a profile of menus $\bar{\boldsymbol{m}}$ such that for every i,*

$$\bar{m}_i \in \underset{m_i\in\mathcal{C}^N}{\arg\max} \;\; \Pi_i(m_i, \bar{m}_{-i}, \boldsymbol{t}(m_i, \bar{m}_{-i}))$$

In words, for a given risk adjustment scheme, a Nash equilibrium in pure strategies consists of a profile of menus such that each insurer's menu of contracts is a best response to the remaining insurers' menus of contracts. Because I only consider pure strategy Nash equilibria, henceforth, I call a Nash equilibrium in pure strategies simply an equilibrium.

I impose the following straightforward restriction on the set of feasible risk adjustment schemes.

Definition 3. $\boldsymbol{t}(\boldsymbol{m})$ *is feasible if and only if* $t_i(m_o, m_{-i}) = 0$ *for every* i, m_{-i}, *and* $\sum_i t_i(\boldsymbol{m}) \geq 0$.

In other words, a risk adjustment scheme is feasible if and only if an insurer offers a menu of contracts that contains only the null contracts; its transfers are zero regardless of the menus offered by the remaining insurers. This allows the insurers to access their outside option, i.e., zero, if they wish. Second, it imposes a *budget balance* condition by requiring the sum of the transfers paid to weakly exceed the sum of the transfers received. In that sense, the risk adjustment scheme is purely redistributive and does not require tax subsidies. Hereafter, I concentrate solely on risk adjustment schemes that satisfy Definition 3.

A useful preliminary result is the following:

Lemma 1. *Let* $\bar{\boldsymbol{m}}$ *be an equilibrium profile; then, for every* $c_1, c_2 \in \arg\max_{c \in \Gamma(\bar{\boldsymbol{m}})} U^\theta(c)$, $c_1 = c_2$.

Proof. See the appendix. □

Intuitively, the result is a consequence of the strict concavity and strict monotonicity of the utility function. For every two distinct contracts between which a consumer of a certain type is indifferent, there is a convex combination that she strictly prefers and yields an insurer strictly higher profits. This makes asymmetric equilibria impossible to sustain. Thanks to Lemma 1, one can uniquely define by $\boldsymbol{c}(\bar{\boldsymbol{m}})$ an incentive compatible allocation associated with equilibrium $\bar{\boldsymbol{m}}$ such that for every θ, $c^\theta(\bar{\boldsymbol{m}}) \in \arg\max_{c \in \Gamma(\bar{\boldsymbol{m}})} U^\theta(c)$, and call $\boldsymbol{c}(\bar{\boldsymbol{m}})$ an *equilibrium allocation*.

5 Optimal Risk Adjustment

The question of interest is whether a risk adjustment scheme exists that discourages risk selection without restricting efficient market competition. In this section, I characterise such a straightforward and intuitive risk adjustment scheme. Consider, therefore, the following profile of transfers $\hat{\boldsymbol{t}}(\boldsymbol{m})$, such that, for every i,

$$\hat{t}_i(\boldsymbol{m}) = \begin{cases} \Phi_i(\boldsymbol{m}), & \text{if } \Phi_i(\boldsymbol{m}) > 0 \text{ and } \Phi_j(\boldsymbol{m}) < 0 \text{ for some } j \in I - i \\ \frac{\sum_{j \in \{j' : \Phi_{j'}(\boldsymbol{m}) \geq 0\}} \Phi_j(\boldsymbol{m})}{\#\{j' : \Phi_{j'}(\boldsymbol{m}) < 0\}}, & \text{if } \Phi_i(\boldsymbol{m}) < 0 \text{ and } \Phi_j(\boldsymbol{m}) > 0 \text{ for some } j \in I - i \\ 0, & \text{otherwise} \end{cases}$$

Based on all of the menus in the market and market demand, insurer i makes a transfer equal to its profits, if at least one other insurer incurs losses and the former earns strictly positive profits. In any other case, insurer i pays zero. Insurer i receives as a subsidy an equal share of the sum of the insurers' profits (obtained only from those insurers that realise positive profits) if it realises strictly negative profits and at least one other insurer realises strictly positive profits.

The risk adjustment scheme described here is purely redistributive and only engages in transfers to and from the regulator based on the realised profits of insurers. It is rather straightforward to verify that $\hat{\boldsymbol{t}}(\boldsymbol{m})$ satisfies Definition 3.

The following theorem is the main result of the paper.

Theorem 1. *Let the risk adjustment scheme be given by $\hat{\boldsymbol{t}}(\boldsymbol{m})$; (i) if $\bar{\boldsymbol{m}}$ is an equilibrium profile, then $\boldsymbol{c}(\bar{\boldsymbol{m}})$ is efficient, and (ii) every efficient allocation can be supported as an equilibrium allocation.*

Proof. See the appendix. □

The intuition behind this result is as follows. Suppose that efficiency requires cross-subsidisation, and let insurers offer an efficient allocation as a menu of contracts. This is all that is required to show is that no insurer has a profitable deviation. If the menu of contracts offered by all insurers is an efficient allocation, there are three possibilities. First, an insurer can offer a menu that strictly improves the payoff of all types. Second, an insurer can offer a menu that attracts a subset of low-cost types. Third, an insurer can offer a menu that attracts only high-cost types. In the first case, the deviating insurer earns negative profits before transfers, while its rivals realise zero profits before transfers. By the definition of

$\hat{\boldsymbol{t}}(\boldsymbol{m})$, in such a case, the transfer to the deviating insurer is zero, and therefore, she earns strictly negative profits, which renders the deviation unprofitable. In the second case, the deviating insurer definitely harms at least one of the remaining insurers. By the definition of $\hat{\boldsymbol{t}}(\boldsymbol{m})$, the transfer she needs to make to the regulator is equal to her profits. Then, any gains from cream-skimming will be outweighed by the transfer, which also renders the deviation unprofitable. In the third case, the deviating insurer earns strictly negative profits before transfers. By the definition of $\hat{\boldsymbol{t}}(\boldsymbol{m})$, the transfer she needs to earn from the remaining insurers equals their strictly positive profits. However, these profits are less than her losses, which also renders the deviation unprofitable. To see that every equilibrium allocation is efficient, suppose that all insurers offer an inefficient menu of contracts. For every such menu, there exists another menu that attracts all types, i.e., not only the most profitable ones, makes strictly positive profits and does not impose any losses on any of the rivals. Hence, no transfer is imposed. This allows effective market competition in accordance with the theory of efficient competitive markets.

6 Discussion

■ **Ex post vs. Ex ante Risk Adjustment.** Two main risk adjustment schemes are implemented in practice: *ex ante* (or prospective) and *ex post* (or retrospective). In an ex ante risk adjustment scheme, every individual is evaluated (in terms of riskiness) based on observable characteristics (e.g., age, gender, pre-existing conditions, chronic diseases), and the regulator subsidises insurers for every individual they enrol. In an ex post risk adjustment scheme, the regulator subsidises insurers based on the (ex post) realised cost of health care. risk adjustment is a form of ex post risk adjustment.

Although ex post risk adjustment completely eliminates selection motives, whereas ex ante risk adjustment subsidises insurers based on partially informative signals, most health insurance markets rely on ex ante risk adjustment schemes that condition subsidies on ex post realised costs. The prevalent criticism against ex post cost risk adjustment is that it leaves insurers little incentive to bargain with health care providers to reduce prices. In other words, ex post risk adjustment does not promote cost efficiency.[18]

The risk adjustment scheme proposed in this paper seems to circumvent this problem. Because payments are conditioned on ex post realised profits rather than ex post realised costs, insurers maintain their incentive to bargain with health care providers to reduce their prices and attract more customers. To see this, suppose that each insurer were able to reduce the cost of health care at state ω for type θ from ℓ_ω^θ to $\underline{\ell}_\omega^\theta < \ell_\omega^\theta$ at a cost $k_\omega < \underline{\ell}_\omega - \ell_\omega^\theta$. In that framework, it is evident that efficiency would require insurers to engage in bargaining. The question is whether the risk adjustment scheme proposed in this paper would facilitate this. Suppose to the contrary that in equilibrium insurers chose not bargain. Note that by bargaining, an insurer suffers the cost but simultaneously can offer contracts with lower premiums and deductibles that are more attractive to consumers. By arguments similar to those provided in the proof of Theorem 1, one can show that there would be at least one insurer that would have incentive to suffer the cost of bargaining, reduce its premiums and increase its profits by attracting the whole market. Therefore, no such equilibrium would be sustainable. Key is that the risk adjustment scheme taxes and subsidises profits instead of costs.

□ **Extensions.** Numerous extensions of the model seem interesting. First, in many health insurance markets, a minimum coverage (or standardised benefit packages) policy is implemented. This specifies the minimum amount of coverage for every insured individual. This policy is particularly prevalent in markets with compulsory insurance such as the Netherlands or Switzerland. Minimum coverage is usually complemented by individual mandates that specify tax penalties for the uninsured. These policies, alongside open enrolment and community rating, are intended to ameliorate the risk-portfolios of insurers by forcing low-risk individuals to acquire insurance. It is interesting to examine the effect of minimum coverage on the endogenously determined contracts and on efficiency. Preliminary investigations reveal that minimum coverage, alongside community rating and/or asymmetric

[18]The underlying idea is similar to that of regulating a natural monopoly. With rate-of-return regulation, the monopolist has no incentive to engage in R&D towards cost reduction. See, for instance, Laffont and Tirole (1986), Laffont and Tirole (1993), Cabral and Riordan (1989), Cabral (2017).

information, restricts the insurers' action spaces and therefore might be anti-competitive.[19]

Second, in the model I studied, insurers are symmetric competitors in health insurance plans. As a result, they all earn zero profits in equilibrium. An interesting extension is to allow for some asymmetry among insurers such that positive profits are sustained in equilibrium. Although the modelling of imperfect asymmetric competition in screening markets is in a very preliminary stage, recent contributions by Mahoney and Weyl (2014), Veiga and Weyl (2016) and Lester et al. (2015) appear to be a good starting point.[20]

Third, it is interesting to examine equity, alongside efficiency. Policies such as community rating and risk adjustment are intended to reduce income and risk inequality by cross-subsidising low-income from high-income consumers and high-risk from low-risk consumers. In this paper, I did not study equity issues but solely focused on efficiency. It would be interesting to examine a mix of community rating, risk adjustment and minimum coverage to address both equity and efficiency issues. Based on the conjecture that minimum coverage might be anti-competitive, an interesting trade-off between equity and efficiency might arise.

Finally, although in this paper I focused on pure community rating, it might be interesting to consider adjusted community rating, which allows premiums to vary based on demographic characteristics such as age or gender.

□ **Generality of the Results.** The results could extend to a continuum of types and states as in Einav, Finkelstein, and Cullen (2010), Azevedo and Gottlieb (2017) or Veiga and Weyl (2016). However, such an extension would not be as straightforward because, in that case, one would need to employ a measure-theoretic approach.[21] The result could further extend

[19] The intuition is that under community rating or/and adverse selection, incentive constraints need to be satisfied. Minimum coverage restricts the set of feasible contracts and therefore restricts the action space of insurers. With a restricted contract space, insurers may be unable to undercut rivals to dominate the market.

[20] As Chiappori et al. (2006) state, "*...more attention should be devoted to the interaction between imperfect competition and adverse selection on risk aversion. In fact, our theoretical conclusions, together with recent results by Jullien, Salanie, and Salanie (2007), strongly indicate that there is a crying need for such models.*

[21] In Einav, Finkelstein, and Cullen (2010), only one insurance contract is traded in the market and insurers compete on price for this contract. With voluntary participation, allowing only one contract to be traded can lead to results analogous to those in Akerlof (1970). With compulsory insurance, it is questionable whether efficiency is achieved through such a policy. Azevedo and Gottlieb (2017) allow menus of contracts but impose a zero-profit condition per contract. Veiga and Weyl (2016) allow for multi-dimensional heterogeneity but

to environments with four-dimensional heterogeneity, i.e., wealth, cost of care, risk and risk aversion. In other words, one could assume that the (ex post) utility function is state- and type-dependent. Nonetheless, in that case, one would need to consider random contracts as in Prescott and Townsend (1984) or, more recently, Citanna and Siconolfi (2016). This is because with deterministic contracts, the usual arguments underlying the construction of menus that "undercut" rivals, leading to higher profits, are no longer possible. Random contracts linearise, and hence convexify, the action space, allowing for such "undercutting" arguments. Note however that the problem is not related to the existence of the transfers but to the non-convexity of the action space.

7 Conclusion

Risk selection in health insurance markets is a fundamental concern of policy makers worldwide. Inevitably, it has attracted considerable attention in the literature since the early developments of the field. In this article, I contributed to this literature by identifying a negative externality that the insurers impose on one another when efficiency requires cross-subsidisation. I showed that risk selection can be eliminated by inducing insurers to internalise this negative externality. This could be achieved through a simple risk adjustment scheme that depends on their realised profits (losses). I proved the generic existence of an equilibrium, and I showed that every equilibrium is efficient.

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impose the restriction that insurers can offer only one contract.

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APPENDIX A: PROOFS

■ **Preliminary Results.** I provide some preliminary results regarding the set of incentive compatible allocations that facilitate the proofs to follow.

Lemma 2. *For every incentive compatible and individually rational allocation $\boldsymbol{c}$ with $Z(\boldsymbol{c}) > 0$, there exists an incentive compatible and individually rational allocation $\tilde{\boldsymbol{c}}$ such that $U^\theta(\tilde{c}^\theta) > U^\theta(c^\theta)$ for every θ and $Z(\tilde{\boldsymbol{c}}) > 0$.*

Proof. Suppose that $\boldsymbol{c}$ is an incentive compatible allocation such that $Z(\boldsymbol{c}) > 0$. Consider allocation $\tilde{\boldsymbol{c}} = (\tilde{c}^\theta)_\theta$, where for every ω and θ,

$$u_\omega(W^\theta - \tilde{d}^\theta_\omega - \tilde{p}^\theta) = \epsilon u_\omega(W^\theta - d^\theta_\omega - p^\theta) + (1-\epsilon)u_\omega(W^\theta) \tag{6}$$

Because $u_\omega(\cdot)$ is strictly concave for every ω, by Jensen's inequality, for every ω and θ,

$$W^\theta - \tilde{d}^\theta_\omega - \tilde{p}^\theta < \epsilon(W^\theta - d^\theta_\omega - p^\theta) + (1-\epsilon)W^\theta \tag{7}$$

This is equivalent to

$$\tilde{d}^\theta_\omega + \tilde{p}^\theta > \epsilon(d^\theta_\omega + p^\theta) \tag{8}$$

Subtracting ℓ^θ_ω, multiplying by π^θ_ω and summing over ω yields

$$\zeta^\theta(\tilde{c}^\theta) > \epsilon\zeta^\theta(c^\theta) - (1-\epsilon)\sum_\omega \pi^\theta_\omega \ell^\theta_\omega \tag{9}$$

Multiplying Eq. (9) by λ^θ and summing over θ yields

$$\sum_\theta \lambda^\theta \zeta^\theta(\tilde{c}^\theta) > \epsilon \sum_\theta \lambda^\theta \zeta^\theta(c^\theta) - (1-\epsilon)\sum_\theta \lambda^\theta \sum_\omega \pi^\theta_\omega \ell^\theta_\omega \tag{10}$$

Because $\boldsymbol{c} = (c^\theta)_\theta$ is incentive compatible and individually rational by definition and due to Eq. (6), the following are true:

$$U^\theta(c^\theta) \geq U^\theta(c^{\theta'}) \quad \forall \quad \theta, \theta'$$

$$U^\theta(c^\theta) \geq \underline{U}^\theta \quad \forall \quad \theta$$

$$U^\theta(\tilde{c}^\theta) = \epsilon U^\theta(c^\theta) + (1-\epsilon)u_\omega(W^\theta) \quad \forall \quad \theta$$

$$U^\theta(\tilde{c}^{\theta'}) = \epsilon U^\theta(c^{\theta'}) + (1-\epsilon)u_\omega(W^\theta) \quad \forall \quad \theta, \theta'$$

Therefore, $\tilde{c}^\theta$ is incentive compatible and individually rational. Moreover, for any

$$\frac{\sum_\theta \lambda^\theta \sum_\omega \pi^\theta_\omega \ell^\theta_\omega}{\sum_\theta \lambda^\theta \sum_\omega \pi^\theta_\omega \ell^\theta_\omega + Z(\boldsymbol{c})} < \epsilon < 1$$

it is true that $U^\theta(\tilde{c}^\theta) > U^\theta(c^\theta)$ for every $\theta \in \Theta$ and $Z(\tilde{\boldsymbol{c}}) > 0$. □

Corollary 1. *If $\boldsymbol{c}$ is an efficient allocation, then $Z(\boldsymbol{c}) = 0$.*

Lemma 3. *Suppose that $\boldsymbol{t}(\boldsymbol{m})$ satisfies Definition 3. If $\bar{\boldsymbol{m}}$ is an equilibrium, then the following must hold:*

(a) $\Pi_i(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}})) \geq 0$ for every i, and

(b) there exists j such that $\Pi_j(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}}))) \leq \sum_i \Pi_i(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}}))$ with the inequality being strict when $\sum_i \Pi_i(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}})) > 0$.

According to Part (a) of Lemma 3, in equilibrium (whenever this exists), each insurer can guarantee itself zero profits. This straightforward result requires no formal proof. It relies on the fact that insurers can access their outside option due to Definition 3. Part (b) of Lemma 3 is a rather immediate consequence of Part (a). Because each insurer makes positive profits in equilibrium, the aggregate industry profits are higher than those of any individual insurer and strictly higher when the aggregate industry profits are strictly positive.

□ **Proof of Lemma 1.** I prove the lemma by contradiction. Let $\bar{\boldsymbol{m}} = (\bar{m}_1, \bar{m}_2)$ be an equilibrium and suppose that there exist η and $c_1, c_2 \in \arg\max_{c \in \Gamma(\bar{\boldsymbol{m}})} U^\eta(c)$ such that $c_1 \neq c_2$. Let $c^\theta(\bar{\boldsymbol{m}})$ denote the contract chosen by type $\theta \neq \eta$ when the profile of menus is $\bar{\boldsymbol{m}}$. Consider contract $\bar{c}^\eta$ such that

$$U^\eta(\bar{c}^\eta) = \frac{\sum_{i \in \{j: c_1 \in \bar{m}_j\}} q_i(c_1|\bar{\boldsymbol{m}})}{\lambda^\eta} U^\eta(c_1) + \frac{\sum_{i \in \{j: c_2 \in \bar{m}_j\}} q_i(c_2|\bar{\boldsymbol{m}})}{\lambda^\eta} U^\eta(c_2) \tag{11}$$

As in Lemma 2, $U^\theta(c^\theta(\bar{\boldsymbol{m}})) \geq U^\theta(\bar{c}^\eta)$ for every $\theta \neq \eta$. The profit of contract $\bar{c}^\eta$ is

$$\zeta^\eta(\bar{c}^\eta) > \frac{\sum_{i \in \{j: c_1 \in \bar{m}_j\}} q_i(c_1|\bar{\boldsymbol{m}})}{\lambda^\eta} \zeta^\eta(c_1) + \frac{\sum_{i \in \{j: c_2 \in \bar{m}_j\}} q_i(c_2|\bar{\boldsymbol{m}})}{\lambda^\eta} \zeta^\eta(c_2)$$

The aggregate profit of allocation $\big((c^\theta(\bar{\boldsymbol{m}}))_{\theta\neq\eta}, \bar{c}^\eta\big)$ is

$$\lambda^\eta\zeta^\eta(\bar{c}^\eta) + \sum_{\theta\neq\eta}\lambda^\theta\zeta^\theta(c^\theta(\bar{\boldsymbol{m}})) >$$

$$\sum_{i\in\{j:c_1\in\bar{m}_j\}} q_i(c_1|\bar{\boldsymbol{m}})\zeta^\eta(c_1) + \sum_{i\in\{j:c_2\in\bar{m}_j\}} q_i(c_2|\bar{\boldsymbol{m}})\zeta^\eta(c_2) + \sum_{\theta\neq\eta}\lambda^\theta\zeta^\theta(c^\theta(\bar{\boldsymbol{m}})) =$$

$$\sum_i \Pi_i(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}}))$$

which from Lemma 3 is greater than zero. As in Lemma 2, there exists an allocation $\tilde{\boldsymbol{c}}$ and $\epsilon \in (0,1)$ such that $U^\theta(\tilde{c}^\theta) > U^\theta(c^\theta(\bar{\boldsymbol{m}}))$ for every θ, and

$$Z(\tilde{\boldsymbol{c}}) > \epsilon\big(\lambda^\eta\zeta^\eta(\bar{c}^\eta) + \sum_{\theta\neq\eta}\lambda^\theta\zeta^\theta(c^\theta(\bar{\boldsymbol{m}}))\big) - (1-\epsilon)\sum_\theta\lambda^\theta\sum_\omega\pi^\theta_\omega\ell^\theta_\omega$$

For any

$$\frac{\sum_i \Pi_i(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}})) + \sum_\theta\lambda^\theta\sum_\omega\pi^\theta_\omega\ell^\theta_\omega}{\lambda^\eta\zeta^\eta(\bar{c}^\eta) + \sum_{\theta\neq\eta}\lambda^\theta\zeta^\theta(c^\theta(\bar{\boldsymbol{m}})) + \sum_\theta\lambda^\theta\sum_\omega\pi^\theta_\omega\ell^\theta_\omega} < \epsilon < 1$$

it is true that

$$Z(\tilde{\boldsymbol{c}}) > \sum_i \Pi_i(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}}))$$

By Definition 2, for every j,

$$\Pi_j(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}})) \geq \Pi_j((\tilde{\boldsymbol{c}}, \bar{m}_{-j}), \boldsymbol{t}(\tilde{\boldsymbol{c}}, \bar{m}_{-j})) = Z(\tilde{\boldsymbol{c}}) > \sum_i \Pi_i(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}}))$$

This contradicts Lemma 3. Q.E.D.

□ **Proof of Theorem 1.** For Part (i) of Theorem 1, consider $\hat{\boldsymbol{t}}(\boldsymbol{m})$. Let $\bar{\boldsymbol{m}}$ be an equilibrium, and suppose that $\boldsymbol{c}(\bar{\boldsymbol{m}})$ is not efficient. By Lemma 3, only equilibria in which all insurers earn positive profits exist. By Definition 1, there exists an allocation $\hat{\boldsymbol{c}}$ such that $U^\theta(\hat{c}^\theta) \geq U^\theta(c^\theta(\bar{\boldsymbol{m}}))$ for every θ with the inequality being strict for some θ and $Z(\hat{\boldsymbol{c}}) \geq 0$. Let $H = \{\theta : U^\theta(\hat{c}^\theta) = U^\theta(c^\theta(\bar{\boldsymbol{m}}))\}$. Consider $\tilde{\boldsymbol{c}}$, where

$$u_\omega(W^\theta - \tilde{d}^\theta_\omega - \tilde{p}^\theta) = \epsilon_1 u_\omega(W^\theta - d^\theta_\omega(\bar{\boldsymbol{m}}) - p^\theta(\bar{\boldsymbol{m}})) + (1-\epsilon_1)u_\omega(W^\theta - \hat{d}^\theta_\omega - \hat{p}^\theta) \tag{12}$$

By replicating the argument of Lemma 2,

$$Z(\tilde{\boldsymbol{c}}) > \epsilon_1 Z(\boldsymbol{c}(\bar{\boldsymbol{m}})) + (1-\epsilon_1)Z(\hat{\boldsymbol{c}}) = \epsilon_1\sum_i \Pi_i(\bar{\boldsymbol{m}}, \hat{\boldsymbol{t}}(\bar{\boldsymbol{m}})) \geq 0 \tag{13}$$

for every $\epsilon_1 \in (0,1]$, where the last equality follows from Corollary 1. Moreover, for every $\epsilon_1 \in (0,1)$, $U^\theta(\tilde{c}^\theta) > U^\theta(c^\theta(\bar{\boldsymbol{m}}))$ for every $\theta \in \Theta - H$ and $U^\theta(\tilde{c}^\theta) = U^\theta(c^\theta(\bar{\boldsymbol{m}}))$ for every $\theta \in H$. Because $c^\theta(\bar{\boldsymbol{m}})$ and $\hat{\boldsymbol{c}}$ are both incentive compatible and individually rational by definition and due to Eq. (6), the following are true:

$$U^\theta(\tilde{c}^\theta) = \epsilon_1 U^\theta(c^\theta(\bar{\boldsymbol{m}})) + (1-\epsilon_1)U^\theta(\hat{c}^\theta) \quad \forall \quad \theta$$

$$U^\theta(\tilde{c}^{\theta'}) = \epsilon_1 U^\theta(c^{\theta'}(\bar{\boldsymbol{m}})) + (1-\epsilon_1)U^\theta(\hat{c}^{\theta'}) \quad \forall \quad \theta, \theta'$$

$$U^\theta(c^\theta(\bar{\boldsymbol{m}})) \geq U^\theta(c^{\theta'}(\bar{\boldsymbol{m}})) \quad \forall \quad \theta, \theta'$$

$$U^\theta(\hat{c}^\theta) \geq U^\theta(\hat{c}^{\theta'}) \quad \forall \quad \theta, \theta'$$

$$U^\theta(\hat{c}^\theta) \geq \underline{U}^\theta \quad \forall \quad \theta$$

$$U^\theta(c^\theta(\bar{\boldsymbol{m}})) \geq \underline{U}^\theta \quad \forall \quad \theta$$

Therefore, $\tilde{\boldsymbol{c}}$ is incentive compatible and individually rational. From Lemma 2, there exists $\tilde{\tilde{\boldsymbol{c}}}$ and $\epsilon_2 \in (0,1]$ such that

$$Z(\tilde{\tilde{\boldsymbol{c}}}) > \epsilon_2 Z(\tilde{\boldsymbol{c}}) - (1-\epsilon_2)\sum_\theta \lambda^\theta \sum_\omega \pi_\omega^\theta \ell_\omega^\theta$$

For any ϵ_1 arbitrarily close to one and ϵ_2 such that

$$\frac{\sum_i \Pi_i(\bar{\boldsymbol{m}}, \hat{\boldsymbol{t}}(\bar{\boldsymbol{m}})) + \sum_\theta \lambda^\theta \sum_\omega \pi_\omega^\theta \ell_\omega^\theta}{Z(\tilde{\boldsymbol{c}}) + \sum_\theta \lambda^\theta \sum_\omega \pi_\omega^\theta \ell_\omega^\theta} < \epsilon_2 < 1$$

it is true that

$$Z(\tilde{\tilde{\boldsymbol{c}}}) > \sum_i \Pi_i(\bar{\boldsymbol{m}}, \hat{\boldsymbol{t}}(\bar{\boldsymbol{m}})) \tag{14}$$

Consider then j and $\tilde{\tilde{m}}_j = \tilde{\tilde{\boldsymbol{c}}}$. The following are true:

$$\Phi_j(\tilde{\tilde{m}}_j, \bar{\boldsymbol{m}}_{-j}) = Z(\tilde{\tilde{\boldsymbol{c}}}) > 0$$

$$\Phi_{-j}(\tilde{\tilde{m}}_j, \bar{\boldsymbol{m}}_{-j}) = 0$$

because insurer j attracts all types when the other insurers offer menus $\bar{m}_{-j}$. Therefore, the profits of the insurers are

$$\Pi_j((\tilde{\tilde{m}}_j, \bar{\boldsymbol{m}}_{-j}), \hat{\boldsymbol{t}}(\tilde{\tilde{m}}_j, \bar{\boldsymbol{m}}_{-j})) = Z(\tilde{\tilde{\boldsymbol{c}}}) + 0 - 0 = Z(\tilde{\tilde{\boldsymbol{c}}})$$

and

$$\Pi_i((\tilde{\tilde{m}}_j, \bar{\boldsymbol{m}}_{-j}), \hat{\boldsymbol{t}}(\tilde{\tilde{m}}_j, \bar{\boldsymbol{m}}_{-j})) = 0 + 0 - 0 = 0 \quad \forall \quad i \neq j$$

due to the specification of the risk adjustment scheme. Due to Eq. (14) and Lemma 3, it is true that

$$\Pi_j((\tilde{\tilde{m}}_j, \bar{m}_{-j}), \hat{\boldsymbol{t}}(\tilde{\tilde{m}}_j, \bar{\boldsymbol{m}}_{-j})) > \sum_i \Pi_i(\bar{\boldsymbol{m}}, \hat{\boldsymbol{t}}(\tilde{\tilde{m}}_j, \bar{\boldsymbol{m}}_{-j})) \geq \Pi_j(\bar{\boldsymbol{m}}, \boldsymbol{t}(\bar{\boldsymbol{m}}))$$

From Definition 2, it is true that

$$\Pi_j((\tilde{\tilde{m}}_j, \bar{m}_{-j}), \hat{\boldsymbol{t}}(\tilde{\tilde{m}}_j, \bar{\boldsymbol{m}}_{-j})) \leq \Pi_j(\bar{\boldsymbol{m}}, \hat{\boldsymbol{t}}(\bar{\boldsymbol{m}}))$$

Therefore, we have a contradiction.

For Part (ii) of Theorem 1, let $\hat{\boldsymbol{t}}(\boldsymbol{m})$ and consider an efficient allocation $\hat{\boldsymbol{c}}$. Suppose that $\hat{m}_i = \hat{\boldsymbol{c}}$ and $q_i^\theta(\hat{c}^\theta|\boldsymbol{m}) = \lambda^\theta/\#I$ for every θ and i. Due to Corollary 1, $\Phi_i(\hat{\boldsymbol{m}}) = 0$ for every i. There exists no j and $m_j \neq \hat{m}_j$ that improves the profit of j. To see this, suppose to the contrary that there exists $\tilde{m}_j \neq \hat{m}_j$ such that

$$\Pi_j((\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}), \hat{\boldsymbol{t}}(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j})) > 0 \tag{15}$$

This implies that there exists a subset of types $\tilde{\Theta} \subseteq \Theta$ such that

$$\max_{c \in \tilde{m}_j} U^\theta(c) > U^\theta(\hat{c}^\theta) \quad \text{for every} \quad \theta \in \tilde{\Theta}$$

and

$$\max_{c \in \tilde{m}_j} U^\theta(c) \leq U^\theta(\hat{c}^\theta) \quad \text{for every} \quad \theta \in \Theta - \tilde{\Theta}$$

Suppose first that $\Phi_i(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) \geq 0$ for every $i \neq j$. From the specification of the risk adjustment scheme, we know that $\hat{t}_i(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) = 0$ for every i. Now consider allocation $\big((\hat{c}^\theta)_{\theta \in \Theta - \tilde{\Theta}}, (\tilde{c}^\theta)_{\theta \in \Theta - \tilde{\Theta}}\big)$, where $\tilde{c}^\theta \in \arg\max_{c \in \tilde{m}_j} U^\theta(c)$. From Eqs. (5) and (15), it is established that the total profit of this allocation is strictly positive, which contradicts that $\hat{\boldsymbol{c}}$ is efficient. Instead, suppose that $\Phi_i(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) < 0$ for at least one $i \neq j$ and $\Phi_j(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) > 0$. From the specification of the risk adjustment scheme, we know that $\hat{t}_j = \Phi_j(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j})$ and for those $i \neq j$ such that $\Phi_i(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) < 0$:

$$\hat{t}_i(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) = \frac{\Phi_j(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j})}{\#\{j' : \Phi_{j'}(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) < 0\}}$$

Consider again allocation $\left((\hat{c}^\theta)_{\theta\in\Theta-\tilde{\Theta}}, (\tilde{c}^\theta)_{\theta\in\Theta-\tilde{\Theta}}\right)$ as defined previously. The total profit of this allocation is

$$\Phi_j(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) + \sum_{i\neq j} \Phi_i(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j})$$

which if strictly positive contradicts that $\hat{\boldsymbol{c}}$ is efficient.

The case in which $\Phi_i(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) < 0$ for at least one $i \neq j$ and $\Phi_j(\tilde{m}_j, \hat{\boldsymbol{m}}_{-j}) < 0$ is straightforward and can never constitute a profitable deviation, and hence, it is omitted. Q.E.D.

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