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L'esprit pionnier

A More General Definition of Equilibrium in Markets with Adverse Selection

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Abstract

I provide a general definition of equilibrium in markets with adverse selection. An equilibrium is defined as a menu of contracts that makes non-negative aggregate profits such that there exists no other menu that includes it as a subset and makes strictly positive aggregate profits. I show that every efficient menu of contracts is also an equilibrium menu of contracts. Furthermore, I characterise a general sufficient condition under which every equilibrium menu of contracts is efficient, restoring that way the First Fundamental Theorem of Welfare Economics. I provide two possible interpretations for this new definition.

KEYWORDS: Adverse selection, equilibrium, existence, efficiency

JEL CLASSIFICATION: D82, D86,

1 INTRODUCTION

Early contributions in information economics highlighted the difficulties in defining in a universal manner a competitive equilibrium in markets with adverse selection. As it is elegantly demonstrated in Rothschild and Stiglitz (1976) (RS), in markets with adverse selection, companies have a strong incentive to exercise non-linear pricing to discriminate the different types of consumers. An equilibrium then has to be defined, instead of a vector of linear prices, in menus of contracts offered by companies that satisfy certain properties. For instance, an equilibrium in RS is simply defined as a menu of contracts that makes non-negative profits such that there exists no other menu of contracts that if introduced can make strictly positive profits. This reduced form of competition is meant to capture the main flavour of perfectly competitive markets, in which entry and exit occur till the profits of companies are competed away. Its simplicity notwithstanding, this definition may be too strong in some environments. In particular, in environments that

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satisfy a weak monotonicity and sorting conditions, no menu of contracts may satisfy this definition, and in environments that do not satisfy these two conditions, one cannot generically prove that an equilibrium exists.[1] This is an especially important issue because a model with no generic existence of equilibrium makes it almost impossible for a researcher to make predictions about the outcome.

Perhaps as expected, following the seminal contribution of RS, weaker definitions of equilibrium in markets with adverse selection appeared in the literature. For instance, Wilson (1977) Miyazaki (1977) and Spence (1978) (WMS) (WMS) define an equilibrium as a menu of contracts that makes non-negative profits, and, there exists no other menu of contracts (the defection) that, if introduced in the market, can make strictly positive profits even if all those contracts that become loss-making after the introduction of the defection are withdrawn from the original menu. The WMS definition can be thought as the reduced-form of a dynamic game in which companies that make losses react by eventually withdraw their loss-making contracts and hence no company could survive if its menu of contracts is not profitable after this reaction. Similarly, Riley (1979) defines an equilibrium as a menu of contracts that makes non-negative profits such that, for every other menu (the defection) that makes strictly positive profits, there is another one (the reaction) that makes strictly positive profits and causes the defection to make losses. Riley's definition differs from that of WMS in the sense that incumbent companies can, instead of exiting the market, react to potential entrants by providing new menus of contracts that attract only the most profitable types, i.e. skim the cream, leaving the entrants making losses.

WMS and Riley prove the existence of an equilibrium in environments in which an RS equilibrium does not exist. Unfortunately, to do so, they solely focus on environments that satisfy the two aforementioned conditions, i.e. weak monotonicity and single crossing. Nonetheless, one can easily see that even in insurance environments with more than two possible states of nature, these two conditions are particularly stringent. Furthermore, their definitions have not yet fully backed-up in game-theoretic grounds.[2] In this

[1] See Dosis (2016) for a formal definition of weak monotonicity and sorting. Loosely speaking, weak monotonicity states that the profit of every contract is always increasing in the same order of types and weak sorting is a weaker condition than single crossing.

[2] Hellwig (1987) studies a three-stage game by allowing companies to offer contracts that can be later withdrawn. Hellwig enables companies to offer at most one contract, he finds that the qualitative features of the equilibrium set are much different than this of the WMS equilibrium. More recent contributions by Mimra and Wambach (2013), Netzer and Scheur (2014) and Diasakos and Koufopoulos (2011) reconfirm the insights of Wilson-Miyazaki-Spence in game-theoretic models but in the rather restricted environment of RS. More specifically, Diasakos and Koufopoulos (2011) extend the game of Hellwig (1987) by allowing firms to commit to the menus of contracts they offer. Netzer and Scheuer (2014) allow companies to offer menus of products and decide whether to stay in the market or not after they observe the menus of products of rivals. To become inactive, a company has to pay an exogenously given withdrawal cost. Mimra and Wambach (2011) allow firms, instead of becoming inactive, to withdraw individual products from those they have offered in an endogenously ending number of rounds. They also examined the case in which new companies can enter the market. To the best of my knowledge, the reactive equilibrium of WMS has not been entirely backed up in an extensive-form game in the most general case of any finite number of types.

Engers and Fernandez (1987) examine a game in which companies can offer menus of contracts in a

paper, I provide a more general definition of equilibrium in markets with adverse selection. An equilibrium is a menu of contracts that makes non-negative aggregate profits such that there exists no other menu that includes it as a subset and makes strictly positive aggregate profits. I show that every efficient menu of contracts is also an equilibrium menu of contracts. Notably, this result holds in every environment, even in those environments that satisfy neither weak monotonicity nor sorting. Furthermore, I characterise a general sufficient condition under which every equilibrium menu of contracts is efficient, restoring that way the First Fundamental Theorem of Welfare Economics. This condition holds even in environments that satisfy neither monotonicity nor sorting. I show that this new definition of equilibrium can be considered as the reduced form a dynamic game provided in Dosis (2016) which makes it particularly appealing.

In Section 2, I describe the model. In Section 3, I give the formal definition of an equilibrium. In Section 4, I prove the two main results of the paper. In Section 5, I provide a brief conclusion.

2 The Model

■ **The General Environment with a Finite Number of Types.** There is a measure one of consumers. Each consumer belongs to a certain class (type). The set of possible types is finite $t = 1, ..., T$. With some abuse of notation, I will also denote as T the set of types. The type of a consumer may include non-observable characteristics such as riskiness, attitude towards risk, income, etc, and/or observable characteristics that cannot be used by companies for discrimination purposes such as gender, income, race, etc.[3] The share of type t consumers in the population is λ^t, with $\sum_t \lambda^t = 1$. Let $\mathcal{X} \subset \mathbb{R}^\Omega$ denote the set of contracts available, with representative element $x \in \mathcal{X}$ and $d : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ be a metric defined on $\mathcal{X}$.[4] $(\mathcal{X}, d)$ defines a metric space and I assume this to be a compact. A consumer of type t has preferences represented by a continuous utility function $U^t : \mathcal{X} \to \mathbb{R}$ defined on $(\mathcal{X}, d)$. The status quo utility of type t is U_0^t and, for simplicity, I assume that $U^t(x) \geq U_0^t$ for every t and $x \in \mathcal{X}$, which simply means that the set of all available contracts that a company can offer are preferred over the status quo utility for all types. The profit function of type t is $\pi^t : \mathcal{X} \to \mathbb{R}$, where $\pi^t(x)$ is simply the profit of contract x if this is bought by type t. The profit is irrespective of the identity of the company who sells the contract.

A *menu of contracts* is a set of contracts, denoted by a Greek small letter such as α, β etc. The set of all available menus of contracts is $2^{|\mathcal{X}|}$; the power set of $\mathcal{X}$. Let $x^t(\alpha)$ be the set of utility maximising contracts for type t that are also profit maximising when the

potentially infinite number of rounds. Similarly to the studies cited above, they find that the qualitative features of the equilibrium set significantly differ from those of Riley.

[3]Depending on institutional details, discrimination based on observable characteristics such as gender, income, race, etc. may be unlawful. For instance, in many insurance markets such as the health insurance market in the US under the recently passed ACA, or the health insurance markets in Switzerland and the Netherlands insurance companies are not allowed to discriminate based on gender or pre-existing conditions.

[4]Note that x usually includes both the price and the technical characteristics of the contract but in order to economise on notation these two together.

menu of contracts is α, or $x^t(\alpha) = \{\arg\max_{x\in\alpha}\ U^t(x)\} \cap \{\arg\max_{x\in\alpha}\ \pi^t(x)\}$. Because a consumer may be indifferent among many different contracts within a menu, to pin down the choice set, I assume that when type-t faces menu α, she always selects $x^t(\alpha)$. Without loss of generality, I assume that $x^t(\alpha)$ is a singleton and with some abuse of notation I denote as $x^t(\alpha)$ the contract that maximises the payoff of type t when facing a menu of contracts α and as $\{x^t(\alpha)\}$ the set whose element is $x^t(\alpha)$. Because part of the contribution of the paper is to examine the efficiency of competitive equilibrium, I now define efficiency.

Definition 2.1. *A menu of contracts α is efficient if and only if: (i) $\sum_t \lambda^t \pi^t(x^t(\alpha)) \geq 0$, and, (ii) there exists no other menu of contracts $\tilde{\alpha}$ such that $\sum_t \lambda^t \pi^t(x^t(\tilde{\alpha})) \geq 0$ and $U^t(x^t(\tilde{\alpha})) \geq U^t(x^t(\alpha))$ for every t with the inequality being strict for at least one t. Let the set of efficient menus of contracts be denoted by $\mathcal{X}^{EFF}$.*

□ **More Special Environments.** In applications, a further structure is usually imposed in the space of admissible utility functions. A commonly-imposed assumption is the following:

Assumption 2.2. *For every menu of contracts $\alpha \notin \mathcal{X}^{EFF}$ such that $\sum_t \lambda^t \pi^t(x^t(\alpha)) \geq 0$, there exists another menu of contracts $\tilde{\alpha} \notin \mathcal{X}^{EFF}$ such that $U^t(x^t(\tilde{\alpha})) > U^t(x^t(\alpha))$ for every t and $\sum_t \lambda^t \pi^t(x^t(\tilde{\alpha})) > 0$. Moreover, for every $\alpha \in \mathcal{X}^{EFF}$, $\sum_{t\in t} \lambda^t \pi^t(x^t(\alpha)) = 0$.*

Assumption (2.2) states that for every menu of contracts that is non-efficient and makes non-negative profits, there exists another menu of contracts that is non-efficient, provides a higher payoff to all types and make strictly positive profits. This assumption is a continuity assumption and, as I show below, is a key to ensuring effective competition.

□ **The Insurance Market.** One of the most prominent examples of the above model is the insurance market in which consumers differ with respect to their risk of suffering state dependent losses. There is a finite set of possible (individual) states $\omega = 1, ..., \Omega$. States are independently distributed among different consumers. Consumer of type t starts with initial wealth W^t and can suffer state-dependent losses $\ell_\omega \geq 0$, with $\ell_1 = 0$ and $\ell_\omega < \ell_{\omega+1}$ for every $\omega = 2, ..., \Omega$. The space of insurance contracts is $\mathcal{X} = \mathbb{R}_+ \times \mathbb{R}_-^{|\Omega|-1}$. In other words a contract specifies accident-dependent benefits, $x = (x_\omega)_\omega$. A consumer of type t has probability p_ω^t to be in state ω with $\sum_{\omega=1}^{\Omega} p_\omega^t = 1$ for every t. The expected utility of type t from insurance contract x is given by $U^t(x) = v_1(W^t - x_1) + \sum_{\omega=2}^{\Omega} p_\omega^t v_\omega(W^t - \ell_\omega + x_\omega)$. The status quo utility of type t is $\underline{U^t} = \sum_{\omega\in\Omega} p_\omega^t v_\omega(W^t - \ell_\omega)$. It is usually assumed that $v_\omega(\cdot)$ is strictly increasing and strictly concave for every ω. If type t buys contract x, then the company who sells this contract makes an expected profit $\pi^t(x) = x_1 - \sum_{\omega=2}^{\Omega} p_\omega^t x_\omega$. Note also that the expected utility index might be state-dependent, unlike most studies in insurance markets where utility function is state-independent. Let us call this model the "general insurance market".

Proposition 2.3. *The general insurance market satisfies Assumption COMP.*

Proof. Suppose that $\alpha \notin \mathcal{X}^{EFF}$ such that $\sum_t \lambda^t \pi^t(x^t(\alpha)) \geq 0$. By Definition (2.1), there exists a menu of contracts $\tilde{\alpha}$ such that $\sum_t \lambda^t \pi^t(x^t(\tilde{\alpha})) \geq 0$ and $U^t(x^t(\tilde{\alpha})) \geq U^t(x^t(\alpha))$ for every t with the inequality being strict for at least one t. Consider the menu of contracts $\bar{\alpha} = \bar{x}_t^t$, where $x^t = (x_\omega)_\omega$ such that $v_1(W^t - \bar{x}_1^t) = qv_\omega(W^t - x_1^t(\alpha)) + (1-q)v_\omega(W^t - x_1^t(\tilde{\alpha}))$ and $v_\omega(W^t - \ell_\omega + \bar{x}_\omega^t) = qv_\omega(W^t - \ell_\omega + x_\omega^t(\alpha)) + (1-q)v_\omega(W^t - \ell_\omega + x_\omega^t(\tilde{\alpha}))$ for every $\omega = 2, .., \Omega, t$. Notice that $U^t(\bar{x}^t) \geq U^t(x^t(\alpha))$ for every t. Because v_ω is strictly concave for every ω, then for every t:

$$W^t - \bar{x}_1^t < q(W^t - x_1^t(\alpha)) + (1-q)(W^t - x_1^t(\tilde{\alpha})) \tag{2.1}$$

and for every ω:

$$W^t - \ell_\omega + \bar{x}_\omega^t < q(W^t - \ell_\omega + x_\omega^t(\alpha)) + (1-q)(W^t - \ell_\omega + x_\omega^t(\tilde{\alpha})) \tag{2.2}$$

(2.1) becomes for every t:

$$\bar{x}_1^t > qx_1^t(\alpha) + (1-q)x_1^t(\tilde{\alpha}) \tag{2.3}$$

and (2.2):

$$-\bar{x}_\omega^t > -qx_\omega^t(\alpha) - (1-q)x_\omega^t(\tilde{\alpha}) \tag{2.4}$$

Multiplying (2.3) by p_1^t and (2.4) by p_ω^t and summing over all ω:

$$\bar{x}_1^t - \sum_{\omega=2}^{\Omega} p_\omega^t \bar{x}_\omega^t > -q(x_1^t(\alpha) - \sum_{\omega=2}^{\Omega} p_\omega^t x_\omega^t(\alpha)) - (1-q)(x_1^t(\tilde{\alpha}) - \sum_{\omega=2}^{\Omega} p_\omega^t x_\omega^t(\tilde{\alpha})) \tag{2.5}$$

The left-hand side of (2.5) is $\pi^t(\bar{x}^t)$ and the right-hand side $q(\pi^t(x^t(\alpha)) - (1-q)(\pi^t(x^t(\tilde{\alpha}))$. If we multiply both sides by λ^t and sum over t then it is evident the aggregate profit of $\bar{\alpha}$ is strictly positive. By repeating the same argument once more, one can show that there exists another menu that makes strictly positive profits and attracts all possible types. □

3 A More General Definition of Equilibrium

■ **The Definition.** In this section, I provide the formal definition of equilibrium.

Definition 3.1. *A menu of contracts α is an equilibrium menu of contracts if and only if: (i) $\sum_{t \in T} \lambda^t \pi^t(x^t(\alpha)) \geq 0$, and (ii) there exists no other menu of contracts $\beta \supset \alpha$ such that $\sum_{t \in T} \lambda^t \pi^t(x^t(\beta)) > 0$. The set of equilibrium menus of contracts is denoted as $\mathcal{X}^{EQ}$.*

In words, an equilibrium consists of a menu of contracts that makes non-negative aggregate profits, and there exists no other menu of contracts that includes the first menu as a subset and makes strictly positive profits.

□ **Justification for the New Definition.** There are two interpretations for this new equilibrium definition. Assume that in a market, incumbent companies operate by offering menus of contracts. Assume then that a new entrant is contemplating to enter the market and offer a new menu of contracts. Then one can think that consumers have costs to switch to new companies and especially if those do not offer the contracts of the incumbent companies. Hence any potential entrant to attract any clients needs to offer at least the menu of contracts that incumbent companies offer.

An alternative interpretation is from a more regulatory perspective. Suppose that a market regulator is mainly concerned about inefficiencies due to adverse selection. Therefore, she decides to pass a regulation according to which new entrants always have to include in their menus the menus that are offered by incumbent companies. In Dosis (2916), I show that there exists a game whose set of equilibrium menus of contracts (or allocations) is equivalent to the one examined in this paper.

4 Properties of Equilibrium

There are two important questions of interest. The first is to examine in what environments an equilibrium exists. The second is whether every equilibrium is efficient. Both issues are fundamental in the study of competitive markets. The first is to help us make predictions about the outcome of the model. The second is, given the First Fundamental Welfare Theorem of Economics, to examine whether the competitive market mechanism indeed produces efficient outcomes. I now turn to the study of these two questions.

Proposition 4.1. *If $\alpha \in \mathcal{X}^{EFF}$, then $\alpha \in \mathcal{X}^{EQ}$.*

Proof. Suppose $\alpha \in \mathcal{X}^{EFF}$. By Definition EFF, $\sum_{t\in T} \lambda^t \pi^t(x^t(\alpha)) \geq 0$ and there exists no other menu $\tilde{\alpha}$ such that $U^t(x^t(\tilde{\alpha})) \geq U^t(x^t(\alpha))$ for every $t \in T$ with the inequality being strict for at least one $t \in T$. Suppose that $\alpha \notin \mathcal{X}^{EQ}$. By Definition EQ, there exists $\beta \supset \alpha$ such that $\sum_{t\in T} \lambda^t \pi^t(x^t(\beta)) > 0$. This means that there exists at least one $t' \in T$ such that $U^{t'}(x^{t'}(\beta)) > U^{t'}(x^{t'}(\alpha))$. Because $\alpha \subset \beta$, it is true that $\min_{x\in\beta} U^t(x^(\beta)) \geq U^t(x^t(\alpha))$ for every $t \in T$. This contradicts the thesis that there exists no other menu $\tilde{\alpha}$ such that $U^t(x^t(\tilde{\alpha})) \geq U^t(x^t(\alpha))$ for every $t \in T$ with the inequality being strict for at least one $t \in T$. □

Proposition 4.2. *If Assumption (2.2) is true and $\alpha \in \mathcal{X}^{EQ}$, then $\alpha \in \mathcal{X}^{EFF}$.*

Proof. Suppose that Assumption (2.2) is satisfied and $\alpha \in \mathcal{X}^{EQ}$. By Definition EQ, there exists no other menu of contracts $\beta \supset \alpha$ such that $\sum_{t\in T} \lambda^t \pi^t(x^t(\beta)) > 0$. Suppose that $\alpha \notin \mathcal{X}^{EFF}$. By Assumption (2.2) and Definition (2.1), there exists another menu of contracts $\tilde{\alpha} \notin \mathcal{X}^{EFF}$ such that $U^t(x^t(\tilde{\alpha})) > U^t(x^t(\alpha))$ for every t and $\sum_t \lambda^t \pi^t(x^t(\tilde{\alpha})) > 0$. Consider now $\tilde{\beta} = \alpha \cup \tilde{\alpha}$. Clearly $\tilde{\beta} \supset \alpha$ and $U^t(x^t(\tilde{\beta})) > U^t(x^t(\alpha))$ for every t and $\sum_t \lambda^t \pi^t(x^t(\tilde{\alpha})) > 0$. This contradicts that there exists no other menu of contracts $\beta \supset \alpha$ such that $\sum_{t\in T} \lambda^t \pi^t(x^t(\beta)) > 0$. □

5 Conclusion

The three most frequently-used definitions of equilibrium in markets with adverse selection are these by Rothschild and Stiglitz (1976), Wilson (1977), Miyazaki (1977) and Spence (1978) and Riley (1979). It is well-known that the definition of Rothschild and Stiglitz (1976) may be too strong in some environments with adverse selection, whereas those by Wilson (1977), Miyazaki (1977) and Spence (1978) and Riley (1979) are weaker but still require considerable structure to show that they are satisfied. In this paper, I provided a more general definition of equilibrium. An equilibrium was defined as a menu of contracts that makes non-negative aggregate profits such that there exists no other menu that includes it as a subset and makes strictly positive aggregate profits. I showed that every (constrained Pareto) efficient allocation satisfies this definition. I then characterised a sufficient condition such that a menu of contracts satisfies the definition of equilibrium only if it is efficient, restoring that way the First Fundamental Theorem of Welfare Economics in markets with adverse selection. I showed that this sufficient condition is satisfied in most insurance environments, even in those that monotonicity or single crossing fail. Lastly, I argued that the definition of equilibrium can be thought as the reduced form of a dynamic game provided in Dosis (2016).

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