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Abstract

I construct an efficient mechanism for competitive markets with adverse selection. In the mechanism, each company offers two menus of contracts: a *public menu* and a *private menu*. The union of all the public menus needs to be offered by every active company in the market. On the contrary, a private menu concerns only the company that offers it. I show that this simple mechanism reduces the set of profitable deviations to the extent that a pure-strategy equilibrium exists in every market with adverse selection. Furthermore, I characterise general, well-studied environments in which the set of equilibrium allocations coincides with the set of efficient allocations.

JEL CLASSIFICATION: D02, D82, D86

KEYWORDS: Adverse Selection, Competition, Mechanism Design, Existence, Efficiency.

1 INTRODUCTION

In their seminal contribution, Rothschild and Stiglitz (1976) (henceforth RS) analysed a competitive market with adverse selection and argued that if companies unrestrictedly compete by offering contracts, an equilibrium may fail to exist. To reduce the set of profitable deviations and guarantee existence, Wilson (1977) restricted the set of contracts that companies are allowed to offer. Although this was not explicitly modelled in an extensive-form game, Wilson (1977) argued that these restrictions should be regarded as the reduced form of a dynamic process in which companies offer contracts that can subsequently be withdrawn if they become unprofitable. Under such restrictions, he showed that an equilibrium exists.[1] Miyazaki (1977) and Spence (1978) extended Wilson's (1977)

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[1] Riley (1979) proposed a related dynamic process according to which companies can add contracts as a response to contracts offered by other companies.

idea by allowing companies to offer menus of contracts instead of single contracts and found that equilibrium not only exists but also is efficient. However, a subsequent contribution by Hellwig (1987) highlighted that the set of equilibrium allocations might not be that envisioned by Wilson (1977), Miyazaki (1977) and Spence (1978) if these dynamics are modelled in an extensive-form game.[2] On the contrary, a plethora of inefficient allocations can be sustained as equilibria. Ever since, a fundamental question has remained unresolved: Is there a game form that restricts competition to guarantee the existence of equilibrium without distorting efficiency?

In this paper, I provide an answer to this fundamental question. I study a market with multiple symmetric companies that offer menus of contracts to privately informed consumers. The distinctive characteristic of the market is that the profit of a contract depends on the identity of the consumer who buys it, as in the seminal contributions of Akerlof (1970), Spence (1973) and RS. Nonetheless, the market I study is more general because it allows for multi-dimensional heterogeneity and does not impose single-crossing. I characterise a fully decentralised mechanism of competition among companies. In the mechanism, each company simultaneously and independently submits two menus of contracts: a *public menu* and a *private menu*. The union of all the public menus needs to be offered by all companies in the market. On the contrary, the private menu consists of contracts that only the company that offered them provides to the consumers. Each consumer selects at most one contract from those that are available in the market from at most one company.

I establish two main results. First, I show that this form of competition reduces the set of profitable deviations to the extent that an equilibrium always exists. I formally prove the result by characterising equilibrium strategies. The main intuition is that if companies set an efficient allocation as a public menu of contracts, then no company can skim the cream by attracting the most profitable consumers, as in RS, without also attracting the least profitable ones. Second, I characterise a wide class of environments in which every equilibrium allocation is necessarily weakly efficient. The idea behind this result is nearly identical to that underlying Bertrand competition: Companies always have an incentive to deviate from a non-efficient outcome and undercut rivals with their private menus to increase their profits. Key to achieving this is the possibility for companies to offer private menus.

Regarding the related literature, scholars have recently exhibited increasing interest in the design of mechanisms for competitive markets with adverse selection. For instance, Asheim and Nilssen (1996) allowed companies to renegotiate their contracts with their customers but prohibit them from discriminating among the different consumer types in the renegotiation stage. Diasakos and Koufopoulos (2011) extended the game of Hellwig (1987) by allowing companies to commit to the menus of contracts they offer and demonstrate that equilibrium exists and is efficient. Netzer and Scheuer (2014) allowed companies to offer menus of contracts and decide whether to remain in the market after having observed their rivals' menus of contracts. To become inactive, companies have

[2]Hellwig (1987) followed the tradition of RS and analysed an environment in which companies are allowed to offer only one contract. Recently, Mimra and Wambach (2011) and Netzer and Scheuer (2014) allowed companies to offer menus of contracts and found that the set of equilibrium allocations is even larger than that in Hellwig (1987), arguing that if companies are allowed to offer contracts that can subsequently be withdrawn, some sort of folk theorem prevails.

to pay an exogenously given withdrawal cost. The existence of equilibrium crucially depends on the exogenous specification of the withdrawal cost. If the cost is high, an equilibrium fails to exist for the same reason that it fails to exist in RS. For certain (small) values of the withdrawal cost, equilibrium exists and is efficient. In a similar vein, Mimra and Wambach (2011) allowed companies, instead of becoming inactive, to withdraw individual contracts among those they have offered in an endogenously ending number of rounds. They showed that, without further restrictions, the equilibrium set of their game contains every incentive compatible and positive profit allocation. They also examined which of these equilibria are robust to entry and showed that only an efficient allocation survives. Finally, Picard (2014) allows companies to offer "participating contracts" such that a consumer who buys a contract needs to "participate" in the profits of the company that offered it. Under this modification, he showed that an equilibrium exists and is efficient.[3]

There are at least two significant differences between the mechanism proposed in this paper and those mentioned above. First, the mechanism proposed in this paper applies to a broader class of environments. Specifically, as I show, for the existence result, no particular assumptions are necessary, and for the efficiency result, the sufficient conditions are considerably less stringent than those satisfied by the insurance environment of RS, whereas all of the aforementioned papers examine the rather restrictive environment of RS. Second, the mechanism proposed in this paper is relatively simpler and only slightly departs from traditional contract competition.

The remainder of the paper is organised as follows. In Section 2, I describe the general environment and a large class of special environments. In Section 3, I describe the mechanism, strategies and efficiency. In Section 4, I state and prove the two main theorems.

2 THE MODEL

■ **The Market with Adverse Selection.** There is a measure one of consumers. Each consumer belongs to a certain class (or type). I denote the set of types by Θ with a representative element $\theta \in \Theta$. The measure of type-θ consumers is λ^θ, where $\sum_{\theta\in\Theta} \lambda^\theta = 1$. There are $i = 1, ..., N$ companies in the market with $N \geq 2$. With some abuse of notation, let N also denote the set of companies. Companies supply contracts. The set of all contracts is denoted by X with a representative element x. I assume that this set is compact under some topology. Type θ has utility function $U^\theta : X \to \mathbb{R}$, which is taken to be continuous for every type. The status quo utility of type θ is $\underline{U}^\theta$. I assume that $U^\theta(x) \geq \underline{U}^\theta$ for every $x \in X$ such that all available contracts improve the payoff of every type. This assumption is only for simplicity and is not important for the results. It is imposed mainly to avoid dealing with participation constraints that unnecessarily complicate the analysis. Every contract is associated with a (net) profit. The profit function for type θ is $\pi^\theta : X \to \mathbb{R}$, which is also assumed to be continuous.

[3] Bisin and Gottardi (2006) and Citanna and Siconolfi (2013) presented results in a similar spirit, albeit in a Walrasian environment.

An *allocation* is a vector of contracts indexed by the set of types, $(x^\theta)_\theta$.[4] An allocation $(x^\theta)_\theta$ is *incentive compatible* if $U^\theta(x^\theta) \geq U^\theta(x^{\theta'})$ for every $\theta, \theta' \in \Theta$.

Definition 2.1. *An allocation $(x^\theta)_\theta$ is strictly efficient if and only if: (i) it is incentive compatible, (ii) $\sum_\theta \lambda^\theta \pi^\theta(x^\theta) \geq 0$, and, (iii) there exists no other allocation $(\tilde{x}^\theta)_\theta$ that satisfies (i) and (ii) and, moreover, $U^\theta(\tilde{x}^\theta) \geq U^\theta(x^\theta)$ for every θ with the inequality being strict for at least one θ. An allocation is weakly efficient if we take all inequalities to be strict in (iii).*

This definition of efficiency is that of standard Pareto efficiency subject to incentive constraints. Note that efficiency is defined with respect to the payoff of the consumers and the average resource constraint.

A class of environments that has been excessively studied in the literature is one in which preferences over contracts for all types admit an expected utility representation.

Assumption 2.2. *$X \subset \mathbb{R}^{|\Omega|}$, $|\Omega| \geq 2$, and for every θ, $U^\theta(x)$, $\pi^\theta(x)$ admit an expected utility representation with*

$$U^\theta(x) = \sum_\omega p_\omega^\theta v_\omega(x_\omega)$$

$$\pi^\theta(x) = \sum_\omega p_\omega^\theta \phi_\omega(x_\omega)$$

where $v_\omega(\cdot)$ is strictly increasing and strictly concave for every $\omega = 1, ..., \Omega$ and $\phi_\omega(\cdot)$ is strictly decreasing and concave for every $\omega = 1, ..., \Omega$.

3 The Mechanism

■ **Description of the Mechanism.** A *menu of contracts* is denoted as m. The set of all possible menus is $X^{|\Theta|} \cup \{\emptyset\}$; the set of all $|\Theta|$-tuples. A *public menu* offered by company i is denoted as m_i^{pu} and a *private* menu as m_i^{pr}. An *action* for company i is denoted as $\alpha_i = (m_i^{pu}, m_i^{pr})$. A company has the right to offer the empty set in either of its menus or even both. I assume that, when $m_i^{pr} = m_i^{pu} = \emptyset$, company i becomes inactive. To simplify the notation as much as possible, I model the payoff from this action through the demand specification.[5] One of the distinctive characteristics of the mechanism is that the union of public menus $\cup_i m_i^{pu}$ needs to be offered by all active companies. An *action profile* is $\boldsymbol{\alpha} = (\alpha_i)_i$. An action profile for all companies but i is $\boldsymbol{\alpha}_{-i} = (...., \alpha_{i-1}, \alpha_{i+1}, ...)$.

Let $Y \subseteq X$. The demand correspondence of type θ when facing the set of contracts Y is denoted as $\xi^\theta(Y) = \arg\max_{x \in Y} \; U^\theta(x)$. It is convenient to define an aggregate demand function for each contract based on the menus of all companies in the market. (Aggregate) Demand by type θ for contract x to company i when the action profile is $\boldsymbol{\alpha}$ is denoted as $d_i^\theta(x, \boldsymbol{\alpha})$. A *demand function* for company i is denoted as $d_i = (d_i^\theta)_\theta$. A *demand profile* is $\boldsymbol{d} = ((d_i^\theta)_\theta)_i$. A demand profile clearly needs to satisfy sequential rationality constraints that I now state.

[4] An allocation defines a mapping from the type space to the set of contracts. In the mechanism design jargon, an allocation is a direct revelation mechanism.

[5] See Definition (3.1) and especially (1) below.

Definition 3.1. *A demand profile* $\boldsymbol{d}$ *is sequentially rational if and only if, for every* i, θ, x *and* $\boldsymbol{\alpha}$*:*

$$(1) \quad d_i^\theta(x,\boldsymbol{\alpha}) = \begin{cases} 0, & \textit{if } m_i^{pr} = m_i^{pu} = \emptyset \\ 0 & \textit{if } x \notin (m_i^{pr} \cup (\cup_j m_j^{pu})) \cap \xi^\theta((\cup_j m_j^{pr}) \cup (\cup_j m_j^{pu})) \\ \delta \in [0,\lambda^\theta], & \textit{if } x \in (m_i^{pr} \cup (\cup_j m_j^{pu})) \cap \xi^\theta((\cup_j m_j^{pr}) \cup (\cup_j m_j^{pu})) \end{cases}$$

and for every $\theta, \boldsymbol{\alpha}$*:*

$$(2) \quad \sum_i \sum_{x \in (m_i^{pr} \cup (\cup_j m_j^{pu})) \cap \xi^\theta((\cup_j m_j^{pr}) \cup (\cup_j m_j^{pu}))} d_i^\theta(x,\boldsymbol{\alpha}) = \lambda^\theta$$

According to (1), the demand for contract x by type θ to company i is zero if company i decides to become inactive, i.e., $m_i^{pr} = m_i^{pu} = \emptyset$, or when this contract is not in the demand correspondence of type θ, i.e., $x \notin (m_i^{pr} \cup (\cup_j m_j^{pu})) \cap \xi^\theta((\cup_j m_j^{pr}) \cup (\cup_j m_j^{pu}))$.[6] It is positive only if this contract is in the demand correspondence of type θ, i.e., $x \in (m_i^{pr} \cup (\cup_j m_j^{pu})) \cap \xi^\theta((\cup_j m_j^{pr}) \cup (\cup_j m_j^{pu}))$. (2) is a market clearing condition. It states that the sum of the demands for all contracts that belong to the demand correspondence of type θ sums to λ^θ, i.e., the ex ante measure of type θ.

The profit of company i by choosing action α_i when all other companies play actions $\boldsymbol{\alpha}_{-i}$ and the demand function is $d_i = (d_i^\theta)_\theta$ is

$$\Pi_i(\alpha_i, \boldsymbol{\alpha}_{-i}|d_i) = \sum_{\theta\in\Theta} \sum_{x \in m_i^{pr} \cup (\cup_i m_i^{pu})} d_i^\theta(x,(\alpha_i,\boldsymbol{\alpha}_{-i}))\pi^\theta(x)$$

Definition 3.2. *A subgame perfect Nash equilibrium consists of a pair of action-demand profiles* $(\bar{\boldsymbol{\alpha}}, \bar{\boldsymbol{d}})$ *such that*

(i) $\bar{\alpha}_i \in \arg\max_{\alpha_i} \ \Pi_i(\alpha_i, \bar{\boldsymbol{\alpha}}_{-i}|\bar{d}_i) \ \forall \ i$

(ii) $\bar{\boldsymbol{d}}$ *is sequentially rational*

In summary, a subgame perfect Nash equilibrium consists of a profile of actions and a demand profile such that (i) the action of each company is optimal given the actions of all other companies and the demand profile, and, (ii) the demand profile is sequentially rational.

□ **Remarks.** The main discrepancy between the mechanism proposed in this paper and this in RS is the opportunity given to the companies to offer public menus of contracts.[7] Public menus play an indispensable role because they, unlike RS, reduce the set of profitable deviations of potential entrants and, as I show below, guarantee the universal existence of equilibrium. To justify their use, one can adopt the following reasoning. A policy

[6] Note that $(\cup_j m_j^{pr}) \cup (\cup_j m_j^{pu})$ is the set of all contracts offered in the market and $\xi^\theta((\cup_j m_j^{pr}) \cup (\cup_j m_j^{pu}))$ is the demand correspondence of type θ. $m_i^{pr} \cup (\cup_j m_j^{pu})$ is the set of contracts offered by company i. Therefore, the intersection of the two, if this is not the empty set, consists of the contracts that belong to the demand correspondence of type θ and are offered by company i.

[7] Note that if companies are only allowed to offer private menus, the mechanism becomes identical to that in RS.

maker (or regulator) is aware of the possible inefficiencies in a market due to information asymmetries, and she is especially concerned about the possibility of cream skimming that leads to destructive competition. She then decides to design an institution to alleviate these inefficiencies. Consequently, she invites all companies to determine a common set of contracts that all active companies in the market need to include in their offered menus of contracts. If, for instance, the relevant market is insurance, then insurance companies mutually select the set of insurance contracts that all companies should offer in the market. In this paper, this decision is modelled by allowing companies to simultaneously propose a public menu of contracts. The regulator also allows the companies to offer any other contracts they wish, i.e., the private menus of contracts.

Note that the regulator needs to have minimal information to implement the mechanism. In particular, she needs only to ensure that the union of all the public menus are included in the offers of all active companies. For this to be possible, the set of public menus has to be public information and verifiable by a court of law. In the following, I provide a few arguments for why this is easily implemented in practice.

Consider the case of private insurance markets. One can unquestionably argue that, in most countries, these markets belong to the group of most severely regulated markets. On top of the usual consumer protection and antitrust regulation, insurance companies are subject to financial and product regulation. The former is meant to guarantee the health and soundness of insurance companies as financial institutions. The latter puts restrictions on the insurance policies companies are allowed to offer. Usually, those plans need to have the pre-approval of insurance commissioners before they are marketed. This form of strict regulation is meant to promote competition among insurance companies and protect consumers from unfair pricing practices.[8] In addition, regulators usually set minimum insurance requirements that provide a lower bound, in terms of financial liability, for every person needs to be insured. For instance, in the US, almost all states run a policy according to which all vehicle holders are obliged to buy a minimum amount of insurance coverage.[9] In the recent Affordable Care Act (ACA), health insurance mandates give the option to each individual to buy a minimum amount of insurance or pay a penalty.[10] Moreover, insurance companies and consumers can now meet and trade insurance plans on online platforms known as Healthcare Exchange Marketplaces (HEMs). In such highly regulated markets, it is clear that the regulator can easily monitor the contracts offered by insurance companies. Hence, the regulator can ensure that the set of public menus are always offered by all insurance companies.

□ **Relation to Mechanism Design.** An alternative, perhaps simpler, mechanism is for

[8]According to the US National Association Insurance Commissioners: "... For personal property-casualty lines, about half of the states require insurers to file rates and to receive prior approval before they go into effect. With the exception of workers, compensation and medical malpractice, commercial property-casualty lines in many states are subject to a competitive rating approach. Under such a system, regulators typically retain authority to disapprove rates if they find that competition is not working...."

[9]This minimum insurance usually takes the form of minimum insurance requirements that specify a maximum amount each person injured in an accident would receive and the total coverage per accident for property damage.

[10]Health insurance mandates are now implemented at a federal level even though these had been already implemented at a state level in Massachusetts.

the regulator to ask all companies to announce the state, i.e., the type space and distribution. If they all make the same announcement, the regulator implements an efficient allocation conditional on the announcement. If at least two companies make different announcements, then the regulator shuts down the market. Unfortunately, it is well known in the implementation literature that this mechanism supports equilibria that do not belong to the social choice rule that the planner wishes to implement.[11] Maskin (1999) characterised the social choice rules that are fully implementable by providing a general but complex mechanism. The advantage of the mechanism proposed in this paper relative to those discussed in the full implementation literature is that it is considerably simpler. Such simple mechanisms were examined, for instance, in Varian (1994) and Maskin and Tirole (1999).

4 Equilibria

■ **Existence of Equilibrium.** The first result concerns the existence of equilibrium.

Theorem 4.1. *Every strictly efficient allocation can be supported as an equilibrium allocation.*

Proof. Consider the following profile of actions $\bar{\boldsymbol{\alpha}}$, where $\bar{\alpha}_i = (\bar{m}, \bar{m})$ and $\bar{m}$ is strictly efficient. Consider also the demand profile $\bar{\boldsymbol{d}} = ((\bar{d}_i^\theta)_\theta)_i$, where $\bar{d}_i^\theta(x, \bar{\boldsymbol{\alpha}}) = \lambda^\theta / N$ for every θ and $x \in \xi^\theta(\bar{m})$ and $\bar{d}_i^\theta(x, \bar{\boldsymbol{\alpha}}) = 0$ for every θ and $x \notin \xi^\theta(\bar{m})$. In other words, if all companies offer the same efficient allocation as both public and private menu of contracts, then consumers are uniformly distributed among them. Because Nash equilibrium concerns only unilateral deviations, it suffices only to specify the demand profile for every possible unilateral deviation. This means that one needs to specify the demand by every type θ for every contract x for every company i for every action profile of the form $(\tilde{\alpha}_i, \bar{\boldsymbol{\alpha}}_{-i})$. Suppose, then, that if $\tilde{m}_i^{pu} \neq \bar{m}$ or $\tilde{m}_i^{pr} \neq \bar{m}$ and at least one of these is different from the empty set, then $\bar{d}_i^\theta(x, (\tilde{\alpha}_i, \bar{\boldsymbol{\alpha}}_{-i})) = \lambda^\theta$ for every $x \in \xi^\theta(\tilde{m}_i^{pu} \cup \tilde{m}_i^{pr} \cup \bar{m})$ and $\bar{d}_i^\theta(x, (\tilde{\alpha}_i, \bar{\boldsymbol{\alpha}}_{-i})) = 0$ for every $x \notin \xi^\theta(\tilde{m}_i^{pu} \cup \tilde{m}_i^{pr} \cup \bar{m})$. In other words, upon a unilateral deviation from one of the companies to a different menu of contracts, all types buy from that company only. Note that this demand profile satisfies Definition (3.1). If all other companies continue to play $(\bar{m}, \bar{m})$, no type is worse off, relative to what the rest of the companies offer, by switching to the deviating company i, as the latter, by definition, offers all the contracts offered by all other companies in the market. Now, note that $\Pi_i(\bar{\alpha}_i, \bar{\boldsymbol{\alpha}}_{-i} | \bar{d}_i) \geq 0$ for every i. This is because, by definition, an efficient allocation makes non-negative profits, and we specified the demand profile such as all types are uniformly distributed among all companies if all the latter offer the same strictly efficient allocation. Consider, therefore, a unilateral deviation by one of the companies. Based on the specification of the demand given above, any menu that attracts at least one of the types will also attract all the other types. Because menu $\bar{m}$ is strictly efficient, any menu that improves the payoff of at least one type must make strictly negative profits. Hence, no company has an incentive to unilaterally deviate, and $(\bar{\boldsymbol{\alpha}}, \bar{\boldsymbol{d}})$ satisfies Definition (3.2). □

[11] In fact, there are two types of equilibria in this mechanism. First, there are equilibria in which companies fail to coordinate and, hence, the planner shuts down the market. Second, there are equilibria in which companies coordinate in announcing a false state and, hence, the planner implements an incorrect allocation.

The idea behind Theorem (4.1) is that, by construction, the public menu offered by one company must be offered by all other companies. Therefore, if all companies offer an efficient allocation as a public menu, then no company can profitably "skim the cream" in the market, i.e., attract the relatively high-profitable types, without also attracting some of the low-profitable types (as in RS). This very fact renders any deviation unprofitable. Note that in the proof of Proposition (4.1), I assumed that following a unilateral deviation, all types buy a contract with the deviating company. Despite this demand profile being sequentially rational, given that no consumer becomes worse off by switching to the deviating company as the latter, by definition, offers the contracts offered by all other companies, it is clearly an extreme case. All the proof required is that the deviating company attracts a large number of all types in the market. An intuitive behavioural explanation for this off-the-equilibrium path strategies is that when consumers observe a deviation from some efficient menu, they expect the rest of the companies in the market to go bankrupt. As such, a large share of all types also switches to the company that sets a different menu of contracts. This includes even types for which the terms of their preferred contract remain the same across all companies.

Finally, note that the equilibrium strategies characterised above are definitely not unique. Other equilibrium strategies can be supported as subgame perfect Nash equilibria. For instance, consider the well-known Rothschild-Stiglitz allocation (RSA) and assume that this is efficient.[12] In that case, one can construct equilibrium strategies according to which all companies offer as public menus the empty set and as private menus the RSA. As in the original studies of RS, the main difficulty arises when the RSA is not efficient. This is exactly where RS find the famous non-existence result. In that case, the use of public contracts plays a indispensable role because it restricts the set of profitable deviations and, hence, guarantees the universal existence of equilibrium.

□ **Efficiency.** To further characterise the equilibrium set, I now turn to environments that satisfy Assumption (2.2). The second main result follows:

Theorem 4.2. *If Assumption (2.2) is satisfied, then every equilibrium allocation is weakly efficient.*

Proof. Assume that Assumption (2.2) holds. Consider an equilibrium $(\bar{\boldsymbol{\alpha}}, \bar{\boldsymbol{d}})$ and denote as $(\bar{x}^\theta)_\theta$ the resulting equilibrium allocation. I show that $(\bar{x}^\theta)_\theta$ is weakly efficient.

Suppose not. By Definition (2.1), if $(\bar{x}^\theta)_\theta$ is not weakly efficient, then there exists an efficient allocation $(\bar{\bar{x}}^\theta)_\theta$ such that $U^\theta(\bar{\bar{x}}^\theta) > U^\theta(\bar{x}^\theta)$ for every θ and $\sum_\theta \lambda^\theta \pi^\theta(\bar{\bar{x}}^\theta) \geq 0$. Consider the following allocation $(\tilde{x}^\theta)_\theta$ such that $u_\omega(\tilde{x}^\theta_\omega) = q u_\omega(\bar{x}^\theta_\omega) + (1-q) u_\omega(\bar{\bar{x}}^\theta_\omega)$ for every θ and ω. Because u_ω is strictly increasing and strictly concave for every ω, $\tilde{x}^\theta_\omega < q\bar{x}^\theta_\omega + (1-q)\bar{\bar{x}}^\theta_\omega$. On the other hand, because ϕ_ω is strictly decreasing, $\phi_\omega(\tilde{x}^\theta_\omega) > \phi_\omega(q\bar{x}^\theta + (1-q)\bar{\bar{x}}^\theta_\omega)$ Because ϕ_ω is concave, $\phi_\omega(q\bar{x}^\theta + (1-q)\bar{\bar{x}}^\theta_\omega) \geq \phi_\omega(q\bar{x}^\theta) + (1-q)\phi_\omega(\bar{\bar{x}}^\theta_\omega)$. Hence,

(3) $$\phi_\omega(\tilde{x}^\theta_\omega) > q\phi_\omega(\bar{x}^\theta_\omega) + (1-q)\phi_\omega(\bar{\bar{x}}^\theta_\omega)$$

One can easily show that $(\tilde{x}^\theta)_\theta$ is incentive compatible. By definition, $(\bar{x}^\theta)_\theta$ and $(\bar{\bar{x}}^\theta)_\theta$

[12] The RSA is the generalisation of the separating menu of contracts in the original study of RS.

are incentive compatible, or $U^\theta(\bar{x}^\theta) \geq U^\theta(\bar{x}^{\theta'})$ and $U^\theta(\bar{\bar{x}}^\theta) \geq U^\theta(\bar{\bar{x}}^{\theta'})$ for every θ, θ'. Equivalently,

$$(4) \quad \sum_{\omega\in\Omega} p^\theta_\omega u_\omega(\bar{x}^\theta_\omega) \geq \sum_{\omega\in\Omega} p^\theta_\omega u_\omega(\bar{x}^{\theta'}_\omega)$$

$$(5) \quad \sum_{\omega\in\Omega} p^\theta_\omega u_\omega(\bar{\bar{x}}^\theta_\omega) \geq \sum_{\omega\in\Omega} p^\theta_\omega u_\omega(\bar{\bar{x}}^{\theta'}_\omega)$$

Multiplying (4) by q and (5) by $(1-q)$ and adding them, we obtain

$$(6) \quad q\sum_{\omega\in\Omega} p^\theta_\omega u_\omega(\bar{x}^\theta_\omega) + (1-q)\sum_{\omega\in\Omega} p^\theta_\omega u_\omega(\bar{\bar{x}}^\theta_\omega) \geq q\sum_{\omega\in\Omega} p^\theta_\omega u_\omega(\bar{x}^{\theta'}_\omega) + (1-q)\sum_{\omega\in\Omega} p^\theta_\omega u_\omega(\bar{\bar{x}}^{\theta'}_\omega)$$

(6) can be re-written as

$$(7) \quad \sum_{\omega\in\Omega} p^\theta_\omega [q u_\omega(\bar{x}^\theta_\omega) + (1-q)u_\omega(\bar{\bar{x}}^\theta_\omega)] \geq \sum_{\omega\in\Omega} p^\theta_\omega [q u_\omega(\bar{x}^{\theta'}_\omega) + (1-q)u_\omega(\bar{\bar{x}}^{\theta'}_\omega)]$$

Note that the left-hand side of (7) is $U^\theta(\tilde{x}^\theta)$ and the right-hand side $U^\theta(\tilde{x}^{\theta'})$, which holds for every $\theta, \theta' \in \Theta$.

Multiplying both sides of (3) by p^θ_ω and summing over ω, we obtain

$$(8) \quad \pi^\theta(\tilde{x}^\theta) > q\pi^\theta(\bar{x}^\theta) + (1-q)\pi^\theta(\bar{\bar{x}}^\theta)$$

.

Multiplying both sides of (8) by λ^θ and summing over θ, we obtain

$$(9) \quad \sum_\theta \lambda^\theta \pi^\theta(\tilde{x}^\theta) > q\sum_\theta \lambda^\theta \pi^\theta(\bar{x}^\theta) + (1-q)\sum_\theta \lambda^\theta \pi^\theta(\bar{\bar{x}}^\theta)$$

Note, however, that $\sum_\theta \lambda^\theta \pi^\theta(\bar{x}^\theta) = \sum_i \Pi_i(\bar{\alpha}_i, \bar{\boldsymbol{\alpha}}_{-i}|\bar{d}_i)$ and $\sum_\theta \lambda^\theta \pi^\theta(\bar{\bar{x}}^\theta) \geq 0$ by Definition (2.1). It is straightforward to show that $\Pi_i(\bar{\alpha}_i, \bar{\boldsymbol{\alpha}}_{-i}|\bar{d}_i) \geq 0$ for every i (if not, then the company with negative profits would offer the empty set as both menus, which guarantees zero profits). There are then two cases.

If $\sum_\theta \lambda^\theta \pi^\theta(\bar{x}^\theta) > 0$, then there exists j such that $\Pi_j(\bar{\alpha}_j, \bar{\boldsymbol{\alpha}}_{-j}|\bar{d}_j) < \sum_\theta \lambda^\theta \pi^\theta(\bar{x}^\theta)$. Moreover, there exists q small enough such that $\sum_\theta \lambda^\theta \pi^\theta(\tilde{x}^\theta) > \Pi_j(\bar{\alpha}_j, \bar{\boldsymbol{\alpha}}_{-j}|\bar{d}_j)$ for some j.

If $\sum_\theta \lambda^\theta \pi^\theta(\bar{x}^\theta) = 0$, then for every i, $\Pi_i(\bar{\alpha}_i, \bar{\boldsymbol{\alpha}}_{-i}|\bar{d}_i) = 0$. Moreover, for every q, $\sum_\theta \lambda^\theta \pi^\theta(\tilde{x}^\theta) > \Pi_j(\bar{\alpha}_j, \bar{\boldsymbol{\alpha}}_{-j}|\bar{d}_j)$ for some j.

Consider, then, $\tilde{\alpha}_j = ((\tilde{x}^\theta)_\theta, \emptyset)$. Because $U^\theta(\tilde{x}^\theta) > U^\theta(\bar{x}^\theta)$ for every θ and in equilibrium $\bar{\boldsymbol{d}}$ is sequentially rational, $\bar{d}_j(\tilde{x}^\theta, (\tilde{\alpha}_j, \bar{\boldsymbol{\alpha}}_{-i})) = \lambda^\theta$ for every θ. This, however, means that $\Pi_j(\tilde{\alpha}_j, \bar{\boldsymbol{\alpha}}_{-i})|\bar{d}_j) > \Pi_j(\bar{\alpha}_j, \bar{\boldsymbol{\alpha}}_{-i})|\bar{d}_j)$ in both cases, which contradicts the no-unilateral deviation condition of Definition (3.2). □

The intuition behind Theorem (4.2) is as follows: Assume that an equilibrium exists such that the equilibrium allocation is not weakly efficient. This is where the private menus become important. First, one can easily show that either there is at least one company in the industry that makes strictly less profits than the aggregate industry profit, or all companies make zero profits. In both cases, because of Assumption (2.2), one can find a menu of contracts that, if introduced as a private menu by some company, i.e., either the one that makes strictly less than the aggregate industry profits or any company if all make zero profits, attracts all types of consumers and makes profits strictly higher than in the supposed equilibrium. This means that at least one company has a profitable unilateral deviation, which contradicts the Definition (3.2).[13] Hence, no non-weakly efficient allocation can be sustained as an equilibrium allocation.

A last point that deserves further discussion is Assumption (2.2) as it stands. One cannot help but notice that in Assumption (2.2), the utility index is type independent, i.e., consumers do not differ with respect to their risk aversion. If the utility index is type dependent, Theorem (4.2) might not hold if one insists on deterministic contracts. In that case, random contracts need to be considered. A random contract is a lottery with the space of feasible contracts as the support. The main difficulty with type-dependent utility indexes is that, without the use of random plans, one cannot work with the usual certainty equivalents, as I essentially do in the proof of Theorem (4.2), that are incentive compatible and increase profits. The strain lies exactly on the type dependence of the ex post utility function. Random contracts linearise the space of menus of contracts and ensure that the mix of two incentive compatible (random) allocations is itself incentive compatible.

5 Conclusion

Since the seminal contribution of RS, there has been widespread concern among economists regarding whether a competitive market with asymmetric information can sustain efficient outcomes as equilibria. In this paper, I proposed a simple but powerful mechanism to answer this question. I showed that, in that simple mechanism, an equilibrium exists and is efficient under fairly general assumptions.

References

[1] Akerlof, G. The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics 84*, 3 (1970), 488–500.

[2] Asheim, G. B., and Nilssen, T. Non-discriminating renegotiation in a competitive insurance market. *European Economic Review 40*, 9 (1996), 1717–1736.

[3] Bisin, A., and Gottardi, P. Efficient competitive equilibria with adverse selection. *Journal of Political Economy 114*, 3 (2006), 485–516.

[13] Note the direct similarity to traditional Bertrand competition.

[4] CITANNA, A., AND SICONOLFI, P. Incentive efficient price systems in large insurance economies with adverse selection. Research paper, Columbia Business School No. 13-45, 2013.

[5] DIASAKOS, T., AND KOUFOPOULOS, K. Efficient Nash equilibrium under adverse selection. Working paper, SSRN, 2011.

[6] HELLWIG, M. Some recent developments in the theory of competition in markets with adverse selection. *European Economic Review 31*, 1 (1987), 319–325.

[7] MASKIN, E. Nash equilibrium and welfare optimality*. *The Review of Economic Studies 66*, 1 (1999), 23–38.

[8] MASKIN, E., AND TIROLE, J. Unforeseen contingencies and incomplete contracts. *The Review of Economic Studies 66*, 1 (1999), 83–114.

[9] MIMRA, W., AND WAMBACH, A. A game-theoretic foundation for the wilson equilibrium in competitive insurance markets with adverse selection. Working paper, CESifo Series No. 3412, 2011.

[10] MIYAZAKI, H. The rat race and internal labor markets. *The Bell Journal of Economics 8*, 2 (1977), 394–418.

[11] NETZER, N., AND SCHEUER, F. A game theoretic foundation of competitive equilibria with adverse selection. *International Economic Review 55*, 2 (2014), 399–422.

[12] PICARD, P. Participating insurance contracts and the Rothschild-Stiglitz equilibrium puzzle. *The Geneva Risk and Insurance Review 39*, 2 (2014), 153–175.

[13] RILEY, J. G. Informational equilibrium. *Econometrica 47*, 2 (1979), 331–359.

[14] ROTHSCHILD, M., AND STIGLITZ, J. Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *The Quarterly Journal of Economics 90*, 4 (1976), 629–649.

[15] SPENCE, M. Job market signaling. *The Quarterly Journal of Economics 87*, 3 (1973), 355–374.

[16] SPENCE, M. Product differentiation and performance in insurance markets. *Journal of Public Economics 10*, 3 (1978), 427–447.

[17] VARIAN, H. A solution to the problem of externalities when agents are well-informed. *The American Economic Review 84*, 5 (1994), 1278–1293.

[18] WILSON, C. A model of insurance markets with incomplete information. *Journal of Economic Theory 16*, 2 (1977), 167–207.

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