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L'esprit pionnier

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Abstract

In a two-task team project with observable task outcomes, optimal incentives prioritize tasks differently depending on task externalities. When the tasks are independent, Principal follows a decreasing order by placing *more essential task first*. A task is more essential if its failure compromises the overall project's chance of success from a task-specific cutoff level by a greater percentage. This definition has no systematic relations to the variance of task outcomes. In particular, a more risky task can be less essential or more essential.

Under externalities, *essentiality* and *impact* jointly determine the optimal ordering. A task with much higher impact can be performed early even if it is less essential. Optimal task ordering thus raises subtle new issues and forms an integral part in team incentives. Our analysis provides some contrast with recent team incentives results.

JEL Classification: D2, D8

Key Words: sequencing, essential tasks, joint projects, team incentives, externalities in teams

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[*]**Very preliminary.**

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1   Introduction

Innovation is a process of trial and error. A medical research firm launching a project for the discovery of a drug has to worry about not only achieving a cure for a disease, it has to make sure that the drug does not have other adverse medical effects for it to be approved by the FDA. In addition, the take-up cost of the drug has to be reasonable for ultimate business success. Various aspects of a project can thus be compartmentalized into individual *tasks*. Besides the discovery of drugs, similar description applies to any technological discovery.

Barring a natural ordering in a subset of the tasks for the planned project, there could be rooms for flexibility in which order to execute some of the other tasks. For instance, in the drug discovery example a research team may present one of several alternative drug options for FDA approval, but even before one comes to that stage the team may be experimenting in developing alternative viable solutions before presenting its best case scenario to the FDA. If one submission fails to receive the approval, another solution can be presented improving the overall chance of the application's success.

It will be assumed that the project in question will succeed for sure if all component tasks succeed but its chance of success still remains positive if some but not all of the tasks fail. The firm has to contract out these tasks to a group of experts. We approach this exercise as a typical moral hazard in teams problem with sequential tasks. If the task sequence can be freely chosen and task outcomes can be observed as the project progresses, the principal (or the firm) has to decide in which order to undertake the various tasks. We raise a number of scenarios and ask some related questions, as follows:

(i) Some tasks' failure means the overall project's chance of success gets compromised much more than the failure in other tasks. Should the former tasks be labeled as more *essential* in the sense that these should have greater priorities?

(ii) Suppose the tasks are interdependent. Say, success in task 1 improves the chance of task 2's success much more than the other way around. Then task 1 has a greater *impact* than task 2. Which of the two criteria – impact and essentiality – has higher significance in the principal's design of incentives?

(iii) Should the principal place more essential tasks first or the one with bigger impact?

(iv) Also, should a more *risky* task be placed early or late in the order? What are the relations between riskiness, impact and essentiality?

(vi) How do the principal's decisions differ from the social planner's optimal decisions?

In team moral hazard problems with sequential tasks but intermediate task outcomes not observable, Winter (2006) had observed that more essential tasks should be placed towards the end of the production chain. The analysis in this paper broadens the meaning of essential tasks and brings in other considerations that are relevant for task ordering in teams. Under appropriate conditions our results do not always agree with Winter's observation.

2   Basic Framework

Consider a project that involves 2 sequential tasks, carried out by 2 agents. Each agent performs one task. For his task, an agent decides whether to exert effort or not. Any agent's cost of effort for either task is c. Effort decision yields either success or failure. Without effort, task 1 is completed successfully with probability $\alpha_1 \in (0,1)$, but with effort the task succeeds with probability $\beta_1 \in (\alpha_1, 1)$. Similarly, without effort Task 2 only succeeds with probability $\alpha_2 \in (0,1)$ but with effort this probability increases to $\beta_2 \in (\alpha_2, 1)$. Denote the task ordering by (i,j), $(i,j) \in \{(1,2),(2,1)\}$. We assume that agent j (assigned to task j, moving second in the sequence) observes only the outcome of task i and not the effort decision of the first mover.

In addition to the task success probabilities $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, we consider the case where the failure of an agent at his task has an impact on the overall success of the grand project. Specifically, if both tasks are successful then the project ends successfully; if task k fails while task l succeeds, then the project succeeds with probability γ_k; and if both tasks fail, then the project fails. We can then think of task k being more important than task l in that $\gamma_k < \gamma_l$. To illustrate, consider the extreme case of $\gamma_k = 0$: this means that task k is absolutely crucial, as failure in this task leads the whole project to fail.

The principal receives the value v if the project succeeds and 0 if the project fails. The rewards (v_i, v_j) are as follows. If player i (assigned to task i) succeeds in his task, then he receives the reward $v_i(S_i)$, and $v_i(F_i)$ if his task fails. As for player j, if he succeeds in his task, then he receives $v_j(\cdot, S_j)$, and if he fails his reward is $v_j(\cdot, F_j)$. Note that either reward applies regardless of the outcome of the task assigned to the other player.

The project, denoted as game $\mathcal{G}$, proceeds as follows. Player i chooses effort e_i, with which he either successfully completes task i (denoted as outcome S_i), or fails to complete it (outcome F_i). Upon observing S_i, Player j chooses effort $e_j(S_i)$, and upon observing F_i Player j also chooses effort $e_j(F_i)$. The principal aims to set the optimal effort inducements $\boldsymbol{v}$ and the optimal ordering of tasks. Except for their task assignment, both players are identical, so in the following analysis what matters is the ordering of tasks and not the assignment of players to a slot in the sequence. Given this, we can use "task i" and "player i" interchangeably.

Consider two cases, namely Cases A and B. In both Case A and B, Player i exerts effort. The difference is that in Case A, Player j exerts effort regardless of the outcome of Player i's task, whereas in Case B, Player j exerts effort if and only if task i succeeds. In other words, the principal employs a contingent plan for task j in Case B but not in Case A. Denote an ordering with a contingent plan for the second task by $(i,\widehat{j})$, and an ordering without it by (i,j).

Immediately we see that it is optimizing for the principal to pay an agent nothing if he fails. To see this, incentive compatibility requires that, for player i,

$$\beta_1 v_i(S_i, \cdot) + (1 - \beta_i) v_i(F_i, \cdot) - c \geq \alpha_1 v_i(S_i, \cdot) + (1 - \alpha_i) v_i(F_i, \cdot)$$

or that $v_i(S_i, \cdot) - v_i(F_i, \cdot) \geq \frac{c}{\beta_i - \alpha_i}$. At the optimum, this constraint binds. But the cheapest way to satisfy $v_i(S_i, \cdot) - v_i(F_i, \cdot) = \frac{c}{\beta_i - \alpha_i}$ is to have $v_i(F_i, \cdot) = 0$. By similar reasoning, optimality requires $v_j(\cdot, F_j) = 0$ for player j.

<u>Case A</u>. Consider the subgame that begins at Player j's decision node following F_i. At this decision node, Player j decides between $e_j = 1$ and $e_j = 0$. Call this subgame $\mathcal{G}(F_i)$. Recall our assumption that $v_j(\cdot, F_j) = 0$. Given F_i, Player j chooses $e_j = 1$ if and only if

$$\beta_j v_j(F_i, S_j) - c \geq \alpha_j v_j(F_i, S_j) \quad \Rightarrow \quad v_j(F_i, S_j) \geq \frac{c}{\beta_j - \alpha_j}.$$

Thus,

$$v_j^*(F_i, S_j) = \frac{c}{\beta_j - \alpha_j}. \tag{1}$$

Next consider the subgame $\mathcal{G}(S_i)$. Here, Player j chooses $e_j = 1$ if and only if

$$\beta_j v_j(S_i, S_j) - c \geq \alpha_j v_j(S_i, S_j) \quad \Rightarrow \quad v_j(S_i, S_j) \geq \frac{c}{\beta_j - \alpha_j}.$$

Thus,

$$v_j^*(S_i, S_j) = \frac{c}{\beta_j - \alpha_j}. \tag{2}$$

Finally, consider the effort decision of Player i. Recall that we assume $v_i(F_i) = 0$. Player i chooses $e_i = 1$ if and only if

$$\beta_i v_i(S_i) - c \geq \alpha_i v_i(S_i) \quad \Rightarrow \quad v_i(S_i) \geq \frac{c}{\beta_i - \alpha_i}.$$

Thus,

$$v_i^*(S_i) = \frac{c}{\beta_i - \alpha_i}. \tag{3}$$

The optimal rewards $\boldsymbol{v^*}$ satisfy (1), (2), and (3). Player i's expected utility is

$$Eu_i(\boldsymbol{v^*}, (i, j)) = \beta_i v_i^*(S_i) - c = \left[\frac{\alpha_i}{\beta_i - \alpha_i}\right] c > 0 \tag{4}$$

and Player j's expected utility is

$$Eu_j(\boldsymbol{v^*}, (i, j)) = \beta_i \left[\beta_j v_j^*(S_i, S_j) - c\right] + (1 - \beta_i)\left[\beta_j v_j^*(F_i, S_j) - c\right] = \left[\frac{\alpha_j}{\beta_j - \alpha_j}\right] c > 0. \tag{5}$$

The Principal's expected profit is

$$
\begin{aligned}
E\Pi(\boldsymbol{v^*}, (i, j)) &= \beta_i \beta_j \left[v - v_i^*(S_i) - v_j^*(S_i, S_j)\right] + \beta_i(1 - \beta_j)\left\{\gamma_j \left[v - v_i^*(S_i)\right] + (1 - \gamma_j)\left[-v_i^*(S_i)\right]\right\} \\
&\quad + (1 - \beta_i)\beta_j\left\{\gamma_i\left[v - v_j^*(F_i, S_j)\right] + (1 - \gamma_i)\left[-v_j^*(F_i, S_j)\right]\right\} \\
&= \beta_i\left[\beta_j + (1 - \beta_j)\gamma_j\right]v + (1 - \beta_i)\beta_j\gamma_i v - \beta_i\left[\beta_j v_i^*(S_i) + (1 - \beta_j)v_i^*(S_i)\right] \\
&\quad - \beta_i\beta_j v_j^*(S_i, S_j) - (1 - \beta_i)\beta_j\left\{\gamma_i\left[v_j^*(F_i, S_j)\right] + (1 - \gamma_i)v_j^*(F_i, S_j)\right\} \\
&= \beta_i\left[\beta_j + (1 - \beta_j)\gamma_j\right]v + (1 - \beta_i)\beta_j\gamma_i v - \beta_i\left[\frac{c}{\beta_i - \alpha_i}\right] - \beta_i\beta_j\left[\frac{c}{\beta_j - \alpha_j}\right] \\
&\quad - (1 - \beta_i)\beta_j\left[\frac{c}{\beta_j - \alpha_j}\right] \\
&= \beta_i\left[\beta_j + (1 - \beta_j)\gamma_j\right]v + (1 - \beta_i)\beta_j\gamma_i v - \beta_i\left[\frac{c}{\beta_i - \alpha_i}\right] - \beta_j\left[\frac{c}{\beta_j - \alpha_j}\right]. \tag{6}
\end{aligned}
$$

Comment 1: $E\Pi(\boldsymbol{v^*}, (1, 2)) = E\Pi(\boldsymbol{v^*}, (2, 1))$.

<u>Case B</u>. We now determine the optimal rewards $\boldsymbol{v^*}$ and the Principal's expected profit given

these rewards when a contingent plan for j is employed. We assume that $v_j(\cdot, F_j) = 0$. Consider again the subgame $\mathcal{G}(F_i)$. Since $e_j = 0$ involves zero cost, individual rationality together with the aim of implementing zero effort at least cost imply that the reward to the Player j when his task is successfully completed following $e_j = 0$ (and Player i has failed) is

$$v_j^*(F_i, S_j) = 0. \tag{7}$$

Next consider the subgame $\mathcal{G}(S_i)$. Here, Player j chooses $e_j = 1$ if and only if

$$\beta_j \widetilde{v}_j(S_i, S_j) - c \geq \alpha_j \widetilde{v}_j(S_i, S_j) \quad \Rightarrow \quad \widetilde{v}_j(S_i, S_j) \geq \frac{c}{\beta_j - \alpha_j},$$

where we use $\widetilde{v}_j(\cdot, F_j) = 0$. Thus

$$v_j^*(S_i, S_j) = \frac{c}{\beta_j - \alpha_j}. \tag{8}$$

Finally, consider the effort decision of Player i. As with Case A, we use $v_i(F_i) = 0$. Player i chooses $e_i = 1$ if and only if

$$\beta_i \widetilde{v}_i(S_i) - c \geq \alpha_i \widetilde{v}_i(S_i) \quad \Rightarrow \quad v_i(S_i) \geq \frac{c}{\beta_i - \alpha_i}.$$

Thus,

$$v_i^*(S_i) = \frac{c}{\beta_i - \alpha_i}. \tag{9}$$

The optimal rewards $\boldsymbol{v}^*$ satisfy (7), (8), and (9). Player i's expected utility is

$$\mathbb{E}u_i(\boldsymbol{v}^*, (i, \widehat{j})) = \beta_i v_i^*(S_i) - c = \left[\frac{\alpha_i}{\beta_i - \alpha_i}\right] c > 0, \tag{10}$$

while Player j's expected utility is

$$\mathbb{E}u_j(\boldsymbol{v}^*, (i, \widehat{j})) = \beta_i \left[\beta_j v_j^*(S_i, S_j) - c\right] = \left[\frac{\alpha_j \beta_j}{\beta_j - \alpha_j}\right] c > 0. \tag{11}$$

The Principal's expected profit is

$$\begin{aligned}
\mathbb{E}\Pi(\boldsymbol{v}^*, (i, \widehat{j})) &= \beta_i \beta_j \left[v - v_i^*(S_i) - v_j^*(S_i, S_j)\right] + \beta_i(1 - \beta_j)\left\{\gamma_j \left[v - v_i^*(S_i)\right] + (1 - \gamma_j)\left[-v_i^*(S_i)\right]\right\} \\
&\quad + (1 - \beta_i)\alpha_j \left\{\gamma_i \left[v - v_j^*(F_i, S_j)\right] + (1 - \gamma_i)\left[-v_j^*(F_i, S_j)\right]\right\} \\
&= \beta_i \left[\beta_j + (1 - \beta_j)\gamma_j\right] v + (1 - \beta_i)\alpha_j \gamma_i v - \beta_i \left[\beta_j \widetilde{v}_i^*(S_i) + (1 - \beta_j)\widetilde{v}_i^*(S_i)\right] \\
&\quad - \beta_i \left[\beta_j \widetilde{v}_j^*(S_i, S_j)\right] - (1 - \beta_i)\alpha_j \underbrace{\widetilde{v}_j^*(F_i, S_j)}_{=0} \\
&= \beta_i \left[\beta_j + (1 - \beta_j)\gamma_j\right] v + (1 - \beta_i)\alpha_j \gamma_i v - \beta_i \left[\frac{c}{\beta_i - \alpha_i}\right] - \beta_i \beta_j \left[\frac{c}{\beta_j - \alpha_j}\right]. \tag{12}
\end{aligned}$$

The Principal might prefer not to incentivize effort from Player j following failure in task i if the probability that the project succeeds given this failure is too low for the effort cost to be worthwhile. This is easiest to see in the extreme case of $\gamma_i = 0$: failure in task i results in failure of the entire project for certain following which the best action would be to terminate the project.

$E\Pi(v^*, (i, \widehat{j})) > E\Pi(v^*, (i, j))$ if and only if

$$(1 - \beta_i)\gamma_i(\beta_j - \alpha_j)v < (1 - \beta_i)\frac{\beta_j c}{\beta_j - \alpha_j} \tag{13}$$

$$\text{i.e.,} \qquad \gamma_i < \frac{\beta_j c}{(\beta_j - \alpha_j)^2 v} = \bar{\gamma}_i. \tag{14}$$

We see that $\bar{\gamma}_i \leq 1$ if and only if

$$\beta_j c \leq (\beta_j - \alpha_j)^2 v.$$

Comment 2: Let the task ordering be (i, j). Suppose that the player assigned to task i is induced to exert effort, and that the player assigned to task j is induced to exert effort following success by Player i. The Principal prefers not to incentivize effort from Player j following failure by Player i if γ_i falls below $\bar{\gamma}_i$, where $\bar{\gamma}_i = \frac{\beta_j c}{(\beta_j - \alpha_j)^2 v}$.

Fix $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, c and v. We see that $\bar{\gamma}_i$ is increasing in γ_j.

Observation: $\bar{\gamma}_1 - \bar{\gamma}_2 = \frac{c}{v}\left[\frac{\beta_2}{(\beta_2 - \alpha_2)^2} - \frac{\beta_1}{(\beta_1 - \alpha_1)^2}\right]$.

1. If $\frac{\beta_2}{(\beta_2 - \alpha_2)^2} - \frac{\beta_1}{(\beta_1 - \alpha_1)^2} > 0$, then $\bar{\gamma}_1 > \bar{\gamma}_2$.

2. If $\frac{\beta_2}{(\beta_2 - \alpha_2)^2} - \frac{\beta_1}{(\beta_1 - \alpha_1)^2} = 0$, then $\bar{\gamma}_1 = \bar{\gamma}_2$.

3. If $\frac{\beta_2}{(\beta_2 - \alpha_2)^2} - \frac{\beta_1}{(\beta_1 - \alpha_1)^2} < 0$, then then $\bar{\gamma}_1 < \bar{\gamma}_2$.

Proposition 1 *Let the task ordering be (i, j), and define the 'threshold value' of task i to be*

$$\bar{\gamma}_i = \frac{\beta_j c}{(\beta_j - \alpha_j)^2 v}.$$

Effort is exerted in task i, which can either succeed or fail.

(i) *If $\gamma_i \geq \bar{\gamma}_i$ for $i = 1, 2$, then inducing effort in the task j regardless of the outcome of task i is optimal, and $(1, 2) \sim (2, 1)$.*

(ii) *If $\gamma_1 < \bar{\gamma}_1$ and $\gamma_2 \geq \bar{\gamma}_2$, then the optimal ordering is $(1, \widehat{2})$.*

(iii) *If $\gamma_1 \geq \bar{\gamma}_1$ and $\gamma_2 < \bar{\gamma}_2$, then the optimal ordering is $(2, \widehat{1})$.*

(iv) *If $\gamma_i < \bar{\gamma}_i$ for $i = 1, 2$, then the optimal ordering is $(1, \widehat{2})$ provided that*

$$\frac{\bar{\gamma}_1 - \gamma_1}{\bar{\gamma}_2 - \gamma_2} > \left(\frac{\beta_1 - \alpha_1}{1 - \beta_1}\right)\frac{1 - \beta_2}{\beta_2 - \alpha_2}, \tag{15}$$

and the optimal ordering is $(2, \widehat{1})$ provided that

$$\frac{\bar{\gamma}_1 - \gamma_1}{\bar{\gamma}_2 - \gamma_2} < \left(\frac{\beta_1 - \alpha_1}{1 - \beta_1}\right)\frac{1 - \beta_2}{\beta_2 - \alpha_2}. \tag{16}$$

Otherwise, either $(1, \widehat{2})$ or $(2, \widehat{1})$ is optimal.

Corollary 1 *Suppose the two tasks are identical in every respect except for γ_1, γ_2. Moreover, suppose $\gamma_i < \bar{\gamma}_i$, $i = 1, 2$. Then the optimal ordering is $(1, 2)$ if $\gamma_1 < \gamma_2$.*

Graphically, if we take the point $(\bar{\gamma}_1, \bar{\gamma}_2)$ and draw a horizontal line as well as a vertical line through it, then the result is a division of the set $[0, 1] \times [0, 1]$ into four sections or subsets, where the top-right section corresponds to Prop. 1(i), the top-left section to Prop. 1(ii), the lower-right section to Prop. 1(iii), and the lower-left section to Prop. 1(iv).

From Proposition 1 we see then that for task i to have an impact (i.e., task i's outcome influences the desired effort in task j), it must be that $\gamma_i < \bar{\gamma}_i$; otherwise, effort is incentivized in task j whether task i succeeds or not. One insight is that γ_1 being less than γ_2 does not automatically entail task 1's place ahead of task 2. If both γ values are above their thresholds, the ordering $(1, 2)$ is as good as the ordering $(2, 1)$ though $\gamma_1 < \gamma_2$ (see Example 1 below). In fact, if $\gamma_1 < \gamma_2$ but γ_2 is below its threshold while γ_1 is not, then by Prop. 1(iii) task 2 should go first (Example 2). That said, the fact that a task's γ value is below its threshold is not sufficient for it to be placed first; whether the other task's γ is below its own threshold or not is also taken into account. Specifically, as case (iv) shows, when *both* tasks are below their respective thresholds, then some other factor comes into play, which is how far below one's threshold a task's γ value is relative to the other task.

To see this last point, consider again condition (15), which we rewrite here as

$$\frac{\bar{\gamma}_1 - \gamma_1}{\bar{\gamma}_2 - \gamma_2} > k_j,$$

with k_j denoting the right-hand side of (15). Given that both γ values are below their threshold values, task 1 goes first in the optimal task ordering if and only if this inequality holds. To illustrate, suppose that α and β probabilities are such that $k_j = 1$, and that $\gamma_1 < \gamma_2$. Then the inequality holds (thus the optimal task ordering is $(1, 2)$) if and only if $\gamma_2 - \gamma_1 > \bar{\gamma}_2 - \bar{\gamma}_1$. One can easily imagine other values k_j for which (15) fails to hold although $\gamma_1 < \gamma_2$. Satisfying condition (15) given a larger k_j entails a steeper drop in γ_1 from $\bar{\gamma}_1$.

Example 1. Suppose that $(\alpha_1, \beta_1) = (0.2, 0.4)$, $(\alpha_2, \beta_2) = (0.6, 0.7)$, $c = 1$, and $v = 80$. Then

$$\bar{\gamma}_1 = 0.875, \quad \bar{\gamma}_2 = 0.125, \quad \text{and} \quad k_j = 1.$$

Figure 1 shows the optimal task ordering for any $(\gamma_1, \gamma_2) \in [0, 1] \times [0, 1]$ given these parameter values. The task having the lower γ value need not be placed first in the task sequence. To illustrate, consider the area under the $45°$ line, where $\gamma_2 < \gamma_1$ for any (γ_1, γ_2). In the gray portion, γ_1 falls below its threshold while γ_2 does not, so task 1 is placed first although $\gamma_2 < \gamma_1$. Also note the olive green parallelogram in the bottom of the figure; here γ_2 is below its threshold value, however so is γ_1, with the latter sufficiently below $\bar{\gamma}_1$ for condition (15) to hold, resulting in task 2 going last instead of first in the optimal task ordering. ‖

■ **Comparative statics**

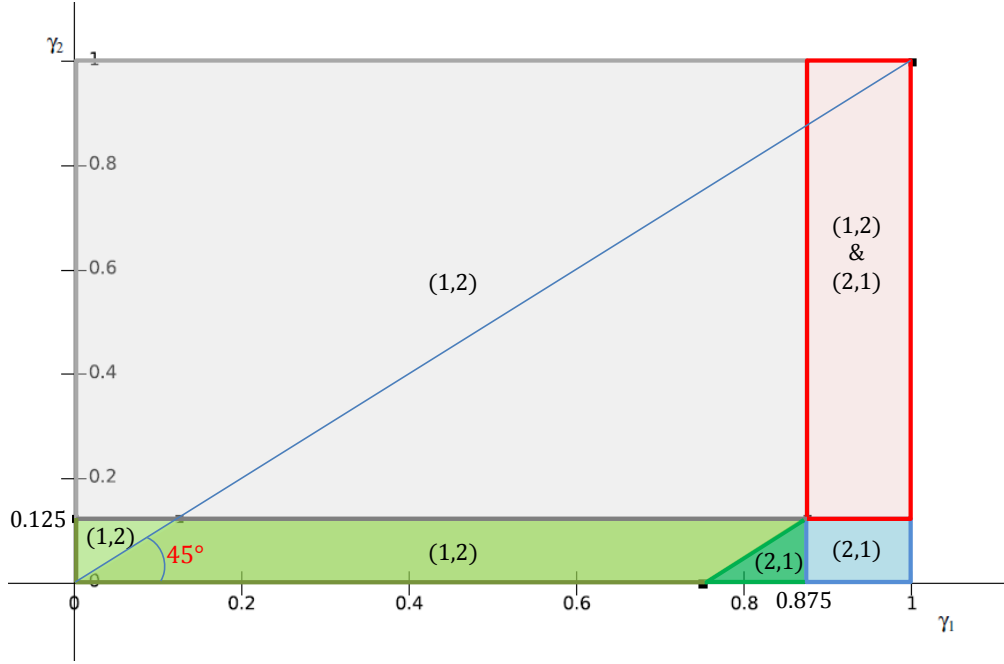**A change in β_j and a reversal of the optimal preference ordering**. Denote the initial

Figure 1: Example 1

optimal task ordering by $(i,j)^*$. We see that

$$\frac{\partial \bar{\gamma}_i}{\partial \beta_j} = -\frac{(\beta_j + \alpha_j)c}{(\beta_j - \alpha_j)^3 v} < 0,$$

while

$$\frac{\partial \bar{\gamma}_j}{\partial \beta_j} = 0.$$

Therefore, an increase in β_j (task ordered last in the optimal task ordering) decreases $\bar{\gamma}_i$ (threshold value of the task placed first in the optimal task ordering) with no accompanying change in $\bar{\gamma}_j$. Let the new threshold value of γ_i be $\bar{\bar{\gamma}}_i$, and suppose that $\gamma_j < \bar{\gamma}_j$ while $\bar{\bar{\gamma}}_i \le \gamma_i < \bar{\gamma}_i$. Then, with everything else constant, an increase in β_j reverses the optimal ordering. Note that this may also result from an decrease in α_j, since

$$\frac{\partial \bar{\gamma}_i}{\partial \alpha_j} = \frac{2\beta_j c}{(\beta_j - \alpha_j)^3 v} > 0,$$

while

$$\frac{\partial \bar{\gamma}_j}{\partial \alpha_j} = 0.$$

■ **Social planner's problem.** Suppose that efforts were observable and enforceable. We show that the information asymmetry makes effort more costly to implement for the principal compared to the social planner, thus the latter would pursue later tasks more often than the principal. To see this, consider the case when task i has been successful. The social planner would push for effort in task j whenever the gain from effort does not fall below its cost, which

in the social planner's case would be c. That is,

$$\{[\beta_j + (1 - \beta_j)\gamma_j] - [\alpha_j + (1 - \alpha_j)\gamma_j]\}v \geq c \quad \Rightarrow \quad (\beta_j - \alpha_j)(1 - \gamma_j)v - c \geq 0.$$

However, in the same situation the principal would induce effort if and only if

$$(\beta_j - \alpha_j)(1 - \gamma_j)v - \beta_j v_j^*(S_i, S_j) = (\beta_j - \alpha_j)(1 - \gamma_j)v - \beta_j \frac{c}{\beta_j - \alpha_j} \geq 0.$$

Clearly, if the principal finds it desirable to incentivize effort, then the social planner should do so as well (since $\frac{\beta_j}{\beta_j - \alpha_j 1} > 1$). But the converse need not be true.

The social planner seeks to maximize the surplus from implementing a particular task ordering. Any task ordering entails effort from the worker assigned to the first task; effort from the worker assigned to the second task may be contingent on first task success or not. We denote the surplus from a task ordering in which second-task effort is not contingent upon first-task outcomes as $S(i, j)$, and the surplus when it is contingent on first-task success as $S(i, \widehat{j})$.

$$S(i, j) = \beta_i[\beta_j + (1 - \beta_j)\gamma_j]v + (1 - \beta_i)\beta_j\gamma_i v - 2c \tag{17}$$

$$S(i, \widehat{j}) = \beta_i[\beta_j + (1 - \beta_j)\gamma_j]v + (1 - \beta_i)\alpha_j\gamma_i v - c - \beta_i c \tag{18}$$

Comment: $S(1, 2) = S(2, 1)$.

$S(i, \widehat{j}) > S(i, j)$ if and only if

$$(1 - \beta_i)\gamma_i(\beta_j - \alpha_j)v < (1 - \beta_i)c \tag{19}$$

$$\text{i.e.,} \qquad \gamma_i < \frac{c}{(\beta_j - \alpha_j)v} = \bar{\gamma}_i^{SP} < \frac{\beta_j}{(\beta_j - \alpha_j)}\left[\frac{c}{(\beta_j - \alpha_j)v}\right] = \bar{\gamma}_i. \tag{20}$$

Proposition 2 *Let the task ordering be* (i, j)*, and define the 'threshold value' of task* i *for the social planner to be*

$$\bar{\gamma}_i^{SP} = \frac{c}{(\beta_j - \alpha_j)v}.$$

Effort is exerted in task i*, which can either succeed or fail.*

(i) *If* $\gamma_i \geq \bar{\gamma}_i^{SP}$ *for* $i = 1, 2$*, then inducing effort in the task* j *regardless of the outcome of task* i *is optimal, and* $(1, 2) \sim (2, 1)$*.*

(ii) *If* $\gamma_1 < \bar{\gamma}_1^{SP}$ *and* $\gamma_2 \geq \bar{\gamma}_2^{SP}$*, then the optimal ordering is* $(1, \widehat{2})$*.*

(iii) *If* $\gamma_1 \geq \bar{\gamma}_1^{SP}$ *and* $\gamma_2 < \bar{\gamma}_2^{SP}$*, then the optimal ordering is* $(2, \widehat{1})$*.*

(iv) *If* $\gamma_i < \bar{\gamma}_i^{SP}$ *for* $i = 1, 2$*, then the optimal ordering is* $(1, \widehat{2})$ *provided that*

$$\frac{\frac{\beta_2 - \alpha_2}{\beta_2}\bar{\gamma}_1 - \gamma_1}{\frac{\beta_1 - \alpha_1}{\beta_1}\bar{\gamma}_2 - \gamma_2} > \left(\frac{\beta_1 - \alpha_1}{1 - \beta_1}\right)\frac{1 - \beta_2}{\beta_2 - \alpha_2}, \tag{21}$$

and the optimal ordering is $(2, \widehat{1})$ provided that

$$\frac{\frac{\beta_2 - \alpha_2}{\beta_2} \bar{\gamma}_1 - \gamma_1}{\frac{\beta_1 - \alpha_1}{\beta_1} \bar{\gamma}_2 - \gamma_2} < \left(\frac{\beta_1 - \alpha_1}{1 - \beta_1} \right) \frac{1 - \beta_2}{\beta_2 - \alpha_2}. \tag{22}$$

Otherwise, either $(1, \widehat{2})$ or $(2, \widehat{1})$ is optimal.

Corollary 2 *Suppose the two tasks are identical in every respect except for γ_1, γ_2. Moreover, suppose $\gamma_i < \bar{\gamma}_i$, $i = 1, 2$. Then the optimal ordering is $(1, 2)$ if $\gamma_1 < \gamma_2$.*

One case where the principal and the social planner would prefer different task orderings: Suppose that $\gamma_1 < \bar{\gamma}_1^{SP}$ (this implies, given (20), that $\gamma_1 < \bar{\gamma}_1$). Suppose further that $\bar{\gamma}_1^{SP} \leq \gamma_2 < \bar{\gamma}_2$, and condition (16) is satisfied for the principal. Then the optimal tasking ordering for the social planner would be $(1, \widehat{2})$ while the optimal ordering for the principal would be $(2, \widehat{1})$.

■ **Risks and task ordering.** We now look at the implications of task riskiness on the optimal ordering of tasks. Specifically, note that effort increases the probability of task success but does not eliminate it: even with effort there remains some chance that the task will fail. If we take task performance (with effort exertion) to be a continuous random variable $X \sim \mathcal{N}(\mu, \sigma^2)$ and calculate the probability of task success β as $P(X > \bar{x})$ for some $\bar{x}$ in the support of the probability distribution, then we can use the variance of X as a measure of the riskiness of the task, with β in turn reflecting this riskiness. As the comparative statics shows, changes in β may affect the optimal task ordering. Thus we can see the effect of task riskiness, represented by the variance of the task performance X, on the optimal task ordering via its impact on β.

To simplify, suppose that both tasks initially have identical α, β, and γ values. It is easy to see that $\bar{\gamma}_i = \bar{\gamma}_j$. Then by Proposition 1, either task ordering is optimal.

Now suppose that performance in task 1 when effort is exerted is the random variable $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and the performance in task 2 following positive effort is $X_2 \sim \mathcal{N}(\mu_1, \sigma_2^2)$, where and $\mu_1 = \mu_2$ and $\sigma_1^2 > \sigma_2^2$: the cdf F_2 is a mean-preserving spread of F_1. To simplify, denote the common mean by μ.

Recall that $\beta = P(X > \bar{x})$. It is easy to see that

$$\sigma_1^2 = \sigma_2^2 \implies \beta_1 = \beta_2$$

for any cut-off value $\bar{x}$. As explained above, either task ordering is optimal in this case.

Returning to $\sigma_1^2 > \sigma_2^2$, we now consider two cases. In the first case, $\bar{x} < \mu$. Note that

$$\beta = P(X > \bar{x}) = P\left(Z > \tfrac{\bar{x} - \mu}{\sigma}\right).$$

We see that if $\bar{x} < \mu$, then

$$\frac{\partial \left(\frac{\bar{x} - \mu}{\sigma} \right)}{\partial \sigma} > 0.$$

Therefore, in this case $\frac{\bar{x}-\mu}{\sigma_1} > \frac{\bar{x}-\mu}{\sigma_2}$ and

$$P\left(Z > \tfrac{\bar{x}-\mu}{\sigma_1}\right) < P\left(Z > \tfrac{\bar{x}-\mu}{\sigma_2}\right) \Rightarrow \beta_1 < \beta_2.$$

Recall that when the tasks had the same variance, $\beta_1 = \beta_2$, hence given the assumptions on the values of the other parameters $(1,2) \sim (2,1)$. But with an increase in task 1's riskiness, $\beta_1 < \beta_2$. Using our comparative statics, we know that this fall in β_1 increases $\bar{\gamma}_2$ but leaves $\bar{\gamma}_1$ unchanged. Denote this higher threshold by $\bar{\bar{\gamma}}_2$.

Suppose that the (identical) γ values are at least as large as their initial threshold values. If $\bar{\gamma}_2 \leq \gamma_2 < \bar{\bar{\gamma}}_2$, then $(2,1) \succ (1,2)$, otherwise either ranking is optimal. Note that when the increase in task 1's riskiness results in a change in the principal's ranking of orderings, it is task 2 rather than task 1 that is placed ahead.

Now suppose that the γ values fall below their initial thresholds. Recall Proposition 1(iv). Note that when the tasks have the same variance,

$$\frac{\bar{\gamma}_1 - \gamma_1}{\bar{\gamma}_2 - \gamma_2} = \left(\frac{\beta_1 - \alpha_1}{1 - \beta_1}\right) \frac{1 - \beta_2}{\beta_2 - \alpha_2}$$

thus $(1,2) \sim (2,1)$. However, an increase in task 1's riskiness decreases β_1 and increases $\bar{\gamma}_2$. The latter decreases the left-hand side of the expression above while the former decreases the right-hand side. Depending on which side falls faster, we have either $(1,2) \succ (2,1)$ or $(2,1) \succ (1,2)$. Again, task 1, which is riskier, need not be placed first.

Now consider the second case of $\bar{x} > \mu$. Now we see that

$$\frac{\partial\left(\frac{\bar{x}-\mu}{\sigma}\right)}{\partial \sigma} < 0.$$

Therefore, in this case $\frac{\bar{x}-\mu}{\sigma_1} < \frac{\bar{x}-\mu}{\sigma_2}$ and

$$P\left(Z > \tfrac{\bar{x}-\mu}{\sigma_1}\right) > P\left(Z > \tfrac{\bar{x}-\mu}{\sigma_2}\right) \Rightarrow \beta_1 > \beta_2.$$

This time, the increase in riskiness increases β_1, which in turn decreases $\bar{\gamma}_2$; $\bar{\gamma}_1$ is unaffected. Denote this lower threshold by $\bar{\bar{\gamma}}_2$. Suppose that $\gamma_1 \geq \bar{\gamma}_1$ and $\gamma_2 \geq \bar{\gamma}_2$; since $\gamma_2 \geq \bar{\gamma}_2 > \bar{\bar{\gamma}}_2$, one would still have $(1,2) \sim (2,1)$. However, suppose $\gamma_1 < \bar{\gamma}_1$ and $\gamma_2 < \bar{\gamma}_2$. There are two possibilities. The first is $\bar{\bar{\gamma}}_2 \leq \gamma_2 < \bar{\gamma}_2$, in which case $(1,2) \succ (2,1)$ for the principal: the less risky task is put first. The second possibility is $\gamma_2 < \bar{\bar{\gamma}}_2 < \bar{\gamma}_2$; here the optimal ranking would depend on whether (15) or (16) would be satisfied, which is not immediately clear since the increase in β_1 increases the right-hand side of the equality above while the fall in $\bar{\gamma}_2$ simultaneously increases the left-hand side.

Proposition 3 *An increase in the riskiness in one task may or may not result in an optimal task ordering where the riskier task goes first.*

■ **Positive externality of task i success on β_j.** Now suppose that, along with the negative externality of task i's failure on the project's overall chance of success, task i also has a positive

externality on task j. Specifically, task i's success increases the effectiveness of player j's effort, represented here as $\beta_j' > \beta_j$, $\Pr(S_j \mid S_i) = \beta_j'$ and $\Pr(S_j \mid F_i) = \beta_j$.

This time, denote the ordering with a contingent plan for the second task by $(i, \widehat{j'})$ and the ordering without it by (i, j').

<u>Case A$'$</u>. Given F_i, Player j chooses $e_j = 1$ if and only if

$$\beta_j v_j(F_i, S_j) - c \geq \alpha_j v_j(F_i, S_j) \quad \Rightarrow \quad v_j(F_i, S_j) \geq \frac{c}{\beta_j - \alpha_j}.$$

Thus,

$$v_j^*(F_i, S_j) = \frac{c}{\beta_j - \alpha_j}. \tag{1$'$}$$

Next consider the subgame $\mathcal{G}(S_i)$. Here, Player j chooses $e_j = 1$ if and only if

$$\beta_j' v_j(S_i, S_j) - c \geq \alpha_j v_j(S_i, S_j) \quad \Rightarrow \quad v_j(S_i, S_j) \geq \frac{c}{\beta_j' - \alpha_j}.$$

Thus,

$$v_j^*(S_i, S_j) = \frac{c}{\beta_j' - \alpha_j}. \tag{2$'$}$$

Finally, consider the effort decision of Player i. Recall that we assume $v_i(F_i) = 0$. Player i chooses $e_i = 1$ if and only if

$$\beta_i v_i(S_i) - c \geq \alpha_i v_i(S_i) \quad \Rightarrow \quad v_i(S_i) \geq \frac{c}{\beta_i - \alpha_i}.$$

Thus,

$$v_i^*(S_i) = \frac{c}{\beta_i - \alpha_i}. \tag{3$'$}$$

The optimal rewards $\mathbf{v}^*$ satisfy $(1')$, $(2')$, and $(3')$. Player i's expected utility is

$$Eu_i(\mathbf{v}^*, (i, j')) = \beta_i v_i^{**}(S_i) - c = \left[\frac{\alpha_i}{\beta_i - \alpha_i}\right] c > 0 \tag{4$'$}$$

and Player j's expected utility is

$$Eu_j(\mathbf{v}^*, (i, j')) = \beta_i \left[\beta_j' v_j^{**}(S_i, S_j) - c\right] + (1 - \beta_i)\left[\beta_j v_j^{**}(F_i, S_j) - c\right]$$
$$= \beta_i \left[\frac{\alpha_j}{\beta_j' - \alpha_j}\right] + (1 - \beta_i)\left[\frac{\alpha_j}{\beta_j - \alpha_j}\right] > 0. \tag{5$'$}$$

Player j's expected utility is less than his utility without a positive externality (compare $(5')$ with (5)).

The Principal's expected profit is

$$\begin{aligned}
E\Pi(\boldsymbol{v}^*,(i,j')) &= \beta_i\beta_j'\big[v - v_i^*(S_i) - v_j^*(S_i,S_j)\big] \\
&\quad + \beta_i(1-\beta_j')\big\{\gamma_j\big[v - v_i^*(S_i)\big] + (1-\gamma_j)\big[-v_i^*(S_i)\big]\big\} \\
&\quad + (1-\beta_i)\beta_j\big\{\gamma_i\big[v - v_j^*(F_i,S_j)\big] + (1-\gamma_i)\big[-v_j^*(F_i,S_j)\big]\big\} \\
&= \beta_i\big[\beta_j' + (1-\beta_j')\gamma_j\big]v + (1-\beta_i)\beta_j\gamma_i v - \beta_i\big[\beta_j'v_i^*(S_i) + (1-\beta_j')v_i^*(S_i)\big] \\
&\quad - \beta_i\beta_j'v_j^{**}(S_i,S_j) - (1-\beta_i)\beta_j'\big\{\gamma_i\big[v_j^{**}(F_i,S_j)\big] + (1-\gamma_i)v_j^{**}(F_i,S_j)\big\} \\
&= \beta_i\big[\beta_j' + (1-\beta_j')\gamma_j\big]v + (1-\beta_i)\beta_j\gamma_i v - \beta_i\big[\tfrac{c}{\beta_i-\alpha_i}\big] - \beta_i\beta_j'\big[\tfrac{c}{\beta_j'-\alpha_j}\big] \\
&\quad - (1-\beta_i)\beta_j\big[\tfrac{c}{\beta_j-\alpha_j}\big]. \tag{6'}
\end{aligned}$$

Corollary 3 *With positive externalities, the principal that is not employing a contingent plan for the second task may not be indifferent between task orderings. Specifically, with positive externalities* $E\Pi(\boldsymbol{v}^*,(1,2')) = E\Pi(\boldsymbol{v}^*,(2,1'))$ *if and only if*

$$\frac{\frac{\beta_2'}{\beta_2} - 1}{\frac{\beta_1'}{\beta_1} - 1} = \frac{(1-\gamma_1)v + \frac{\alpha_1 c}{(\beta_1-\alpha_1)(\beta_1'-\alpha_1)}}{(1-\gamma_2)v + \frac{\alpha_2 c}{(\beta_2-\alpha_2)(\beta_2'-\alpha_2)}}.$$

With a positive externality, player j's is payoff when he exerts effort and succeeds following S_i is smaller, but he has a greater chance of receiving this smaller reward (since $\beta_j' > \beta_j$). The question is whether the change in expected reward decreases profit. We see that

$$\begin{aligned}
E\Pi(\boldsymbol{v}^*,(i,j')) - E\Pi(\boldsymbol{v}^*,(i,j)) &= \beta_i\big[\beta_j' + (1-\beta_j')\gamma_j\big]v - \beta_i\beta_j'\big[\tfrac{c}{\beta_j'-\alpha_j}\big] + \beta_i\beta_j\big[\tfrac{c}{\beta_j-\alpha_j}\big] \\
&\quad - \beta_i\big[\beta_j + (1-\beta_j)\gamma_j\big]v \\
&= \beta_i\left(\beta_j' - \beta_j\right)\left[(1-\gamma_j)v + \big[\tfrac{\alpha_j}{(\beta_j-\alpha_j)(\beta_j'-\alpha_j)}\big]c\right] > 0. \tag{23}
\end{aligned}$$

Comment: Suppose rewards are such player i contributes and player j contributes regardless of the history. Then the positive externality increases the principal's profit.

The principal's profit increases because the positive externality, by raising β_j to β_j' following success in task i, (1) increases the expected value of the project, given by the term

$$\beta_i\big[\left(\beta_j' - \beta_j\right)(1-\gamma_j)\big]v,$$

and (2) lowers the expected reward paid to player j, resulting in the cost savings

$$\beta_i\left[\frac{\alpha_j\left(\beta_j' - \beta_j\right)}{(\beta_j'-\alpha_j)(\beta_j-\alpha_j)}\right]c.$$

This last expression confirms that

$$\frac{\beta_j}{\beta_j - \alpha_j} > \frac{\beta_j'}{\beta_j' - \alpha_j}.$$

It is also easy to check that

$$\frac{\beta_i}{\beta_i - \alpha_i} > \frac{\beta_i'}{\beta_i' - \alpha_i}.$$

Corollary 4 *Suppose the principal incentivizes all players to contribute and, for player j, to contribute regardless of history. Then $(1, 2) \succ (2, 1)$, if*

$$\frac{\frac{\beta_2'}{\beta_2} - 1}{\frac{\beta_1'}{\beta_1} - 1} > \frac{(1 - \gamma_1)\nu + \left[\frac{\alpha_1}{(\beta_1 - \alpha_1)(\beta_1' - \alpha_1)}\right]c}{(1 - \gamma_2)\nu + \left[\frac{\alpha_2}{(\beta_2 - \alpha_2)(\beta_2' - \alpha_2)}\right]c};$$

and $(2, 1) \succ (1, 2)$ if

$$\frac{\frac{\beta_2'}{\beta_2} - 1}{\frac{\beta_1'}{\beta_1} - 1} < \frac{(1 - \gamma_1)\nu + \left[\frac{\alpha_1}{(\beta_1 - \alpha_1)(\beta_1' - \alpha_1)}\right]c}{(1 - \gamma_2)\nu + \left[\frac{\alpha_2}{(\beta_2 - \alpha_2)(\beta_2' - \alpha_2)}\right]c}.$$

<u>Case B′</u>. We now determine the optimal rewards when the principal implements a contingent plan for player j. We assume that $\nu_j(\cdot, F_j) = 0$. Consider again the subgame $\mathcal{G}(F_i)$. As in the benchmark case,

$$\nu_j^*(F_i, S_j) = 0. \tag{7′}$$

Next consider the subgame $\mathcal{G}(S_i)$. Here, Player j chooses $e_j = 1$ if and only if

$$\beta_j' \widetilde{\nu}_j(S_i, S_j) - c \geq \alpha_j \widetilde{\nu}_j(S_i, S_j) \quad \Rightarrow \quad \widetilde{\nu}_j(S_i, S_j) \geq \frac{c}{\beta_j' - \alpha_j},$$

where we use $\nu_j(\cdot, F_j) = 0$. Thus

$$\nu_j^*(S_i, S_j) = \frac{c}{\beta_j' - \alpha_j}. \tag{8′}$$

Finally, consider the effort decision of Player i. Player i chooses $e_i = 1$ if and only if

$$\beta_i \nu_i(S_i) - c \geq \alpha_i \nu_i(S_i) \quad \Rightarrow \quad \widetilde{\nu}_i(S_i) \geq \frac{c}{\beta_i - \alpha_i}.$$

Thus,

$$\nu_i^*(S_i) = \frac{c}{\beta_i - \alpha_i}. \tag{9′}$$

The optimal rewards $\boldsymbol{\nu}^*$ satisfy $(7′)$, $(8′)$, and $(9′)$. Player i's expected utility is

$$Eu_i(\boldsymbol{\nu}^*, (i, \widehat{j'})) = \beta_i \nu_i^*(S_i) - c = \left[\frac{\alpha_i}{\beta_i - \alpha_i}\right]c > 0, \tag{10′}$$

while Player j's expected utility is

$$Eu_j(\boldsymbol{\nu}^*, (i, \widehat{j'})) = \beta_i \left[\beta_j' \nu_j^*(S_i, S_j) - c\right] = \left[\frac{\alpha_j \beta_j'}{\beta_j' - \alpha_j}\right]c < \left[\frac{\alpha_j \beta_j}{\beta_j - \alpha_j}\right]c = Eu_j(\boldsymbol{\nu}^*, (i, \widehat{j})). \tag{11′}$$

The presence of a positive externality decreases the expected utility of player j, although it still allows him some surplus.

The Principal's expected profit is

$$
\begin{aligned}
E\Pi(\boldsymbol{v}^*, (i, \widehat{j'})) &= \beta_i \beta_j' \big[v - v_i^*(S_i) - v_j^*(S_i, S_j) \big] \\
&\quad + \beta_i (1 - \beta_j') \big\{ \gamma_j \big[v - v_i^*(S_i) \big] + (1 - \gamma_j) \big[- v_i^*(S_i) \big] \big\} \\
&\quad + (1 - \beta_i) \alpha_j \big\{ \gamma_i \big[v - v_j^*(F_i, S_j) \big] + (1 - \gamma_i) \big[- v_j^*(F_i, S_j) \big] \big\} \\
&= \beta_i \big[\beta_j' + (1 - \beta_j') \gamma_j \big] v + (1 - \beta_i) \alpha_j \gamma_i v - \beta_i \big[\beta_j' v_i^*(S_i) + (1 - \beta_j') v_i^*(S_i) \big] \\
&\quad - \beta_i \big[\beta_j' v_j^*(S_i, S_j) \big] - (1 - \beta_i) \alpha_j \underbrace{v_j^*(F_i, S_j)}_{=0} \\
&= \beta_i \big[\beta_j' + (1 - \beta_j') \gamma_j \big] v + (1 - \beta_i) \alpha_j \gamma_i v - \beta_i \big[\tfrac{c}{\beta_i - \alpha_i} \big] - \beta_i \beta_j' \big[\tfrac{c}{\beta_j' - \alpha_j} \big] \qquad (12') \\
&= E\Pi(\boldsymbol{v}^*, (i, j')) - (1 - \beta_i)(\beta_j - \alpha_j) \gamma_i v + (1 - \beta_i) \beta_j \big[\tfrac{c}{\beta_j - \alpha_j} \big] \\
&= E\Pi(\boldsymbol{v}^*, (i, j')) + (1 - \beta_i) \big[\tfrac{\beta_j c}{\beta_j - \alpha_j} - (\beta_j - \alpha_j) \gamma_i v \big].
\end{aligned}
$$

When might a principal prefer to use a contingent plan for task j? $E\Pi(\boldsymbol{v}^*, (i, \widehat{j'})) > E\Pi(\boldsymbol{v}^*, (i, j'))$ if and only if

$$
\gamma_i (\beta_j - \alpha_j) v < \frac{\beta_j c}{\beta_j - \alpha_j} \qquad (13')
$$

$$
\text{i.e.,} \qquad \gamma_i < \frac{\beta_j c}{(\beta_j - \alpha_j)^2 v} = \bar{\gamma}_i' = \bar{\gamma}_i. \qquad (14')
$$

Lemma 1 *The existence of a positive externality of task i on task j does not change $\bar{\gamma}_i$.*

$E\Pi(\boldsymbol{v}^*, (i, \widehat{j'}))$ can also be written as

$$
\begin{aligned}
E\Pi(\boldsymbol{v}^*, (i, \widehat{j'})) &= E\Pi(\boldsymbol{v}^*, (i, j')) + (1 - \beta_i) [\bar{\gamma}_i (\beta_j - \alpha_j) v - (\beta_j - \alpha_j) \gamma_i v] \\
&= E\Pi(\boldsymbol{v}^*, (i, j')) + (1 - \beta_i)(\beta_j - \alpha_j) v [\bar{\gamma}_i - \gamma_i] \qquad (24) \\
&= E\Pi(\boldsymbol{v}^*, (i, j')) + \Delta_i. \qquad (12'')
\end{aligned}
$$

Proposition 4 (Essential & externality) *Suppose that there are positive externalities between the two tasks and that*

$$
E\Pi(\boldsymbol{v}^*, (1, 2')) \geq E\Pi(\boldsymbol{v}^*, (2, 1')). \qquad (25)
$$

Task ordering can be characterized as follows:

(i) *When $\gamma_i \geq \bar{\gamma}_i$ for $i = 1, 2$, the optimal ordering is $(1, 2')$ if (25) holds strictly, while the optimal ordering is either $(1, 2')$ or $(2, 1')$ if (25) holds as an equality.*

(ii) *When $\gamma_1 < \bar{\gamma}_1$ and $\gamma_2 \geq \bar{\gamma}_2$, the optimal ordering is $(1, \widehat{2'})$.*

(iii) *When $\gamma_1 \geq \bar{\gamma}_1$ and $\gamma_2 < \bar{\gamma}_2$, the optimal ordering is $(2, \widehat{1'})$ if $\Delta_2 > E\Pi(\boldsymbol{v}^*, (1, 2')) - E\Pi(\boldsymbol{v}^*, (2, 1'))$, either $(2, \widehat{1'})$ or $(1, 2')$ if $\Delta_2 = E\Pi(\boldsymbol{v}^*, (1, 2')) - E\Pi(\boldsymbol{v}^*, (2, 1'))$, and $(1, 2')$ if $\Delta_2 < E\Pi(\boldsymbol{v}^*, (1, 2')) - E\Pi(\boldsymbol{v}^*, (2, 1'))$.*

(iv) *When $\gamma_i < \bar{\gamma}_i$ for $i = 1, 2$, the optimal ordering is $(2, \widehat{1'})$ if $\Delta_2 - \Delta_1 > E\Pi(\boldsymbol{v}^*, (1, 2')) - E\Pi(\boldsymbol{v}^*, (2, 1'))$, either $(2, \widehat{1'})$ or $(1, \widehat{2'})$ if $\Delta_2 - \Delta_1 = E\Pi(\boldsymbol{v}^*, (1, 2')) - E\Pi(\boldsymbol{v}^*, (2, 1'))$, and $(1, \widehat{2'})$ if $\Delta_2 - \Delta_1 < E\Pi(\boldsymbol{v}^*, (1, 2')) - E\Pi(\boldsymbol{v}^*, (2, 1'))$.*

Proposition 5 *Suppose that there are positive externalities between the two tasks and that*

$$E\Pi(\boldsymbol{v}^*, (2, 1')) > E\Pi(\boldsymbol{v}^*, (1, 2')).$$

Task ordering can be characterized as follows:

(i) *When $\gamma_i \geq \bar{\gamma}_i$ for $i = 1, 2$, the optimal ordering is $(2, 1')$.*

(ii) *When $\gamma_1 < \bar{\gamma}_1$ and $\gamma_2 \geq \bar{\gamma}_2$, the optimal ordering is $(1, \widehat{2'})$ if $\Delta_1 > E\Pi(\boldsymbol{v}^*, (2, 1')) - E\Pi(\boldsymbol{v}^*, (1, 2'))$, either $(2, 1')$ or $(1, 2')$ if $\Delta_1 = E\Pi(\boldsymbol{v}^*, (2, 1')) - E\Pi(\boldsymbol{v}^*, (1, 2'))$, and $(2, 1')$ if $\Delta_1 < E\Pi(\boldsymbol{v}^*, (2, 1')) - E\Pi(\boldsymbol{v}^*, (1, 2'))$.*

(iii) *When $\gamma_1 \geq \bar{\gamma}_1$ and $\gamma_2 < \bar{\gamma}_2$, the optimal ordering is $(2, \widehat{1'})$.*

(iv) *When $\gamma_i < \bar{\gamma}_i$ for $i = 1, 2$, the optimal ordering is $(1, \widehat{2'})$ if $\Delta_1 - \Delta_2 > E\Pi(\boldsymbol{v}^*, (2, 1')) - E\Pi(\boldsymbol{v}^*, (1, 2'))$, either $(1, \widehat{2'})$ or $(2, \widehat{1'})$ if $\Delta_1 - \Delta_2 = E\Pi(\boldsymbol{v}^*, (2, 1')) - E\Pi(\boldsymbol{v}^*, (1, 2'))$, and $(2, \widehat{1'})$ if $\Delta_1 - \Delta_2 < E\Pi(\boldsymbol{v}^*, (2, 1')) - E\Pi(\boldsymbol{v}^*, (1, 2'))$.*

■ **Ordering reversal with positive externality.** We now consider the possibility of a reversal of the optimal ordering when task i's success exerts a positive impact on task j. One situation in which reversal in possible is when $\gamma_1 < \bar{\gamma}_1$ and $\gamma_2 \geq \bar{\gamma}_2$. By Proposition 1, when there are no positive externalities between the two tasks, $(1, \widetilde{2}) \succ (2, 1)$. This results largely depends on the fact that when the principal does not deploy a contingent plan for task j, the two task orderings are equivalent, thus $E\Pi(\boldsymbol{v}^*, (1, \widetilde{2})) > E\Pi(\boldsymbol{v}^*, (1, 2))$ also implies that $E\Pi(\boldsymbol{v}^*, (1, \widetilde{2})) > E\Pi(\boldsymbol{v}^*, (2, 1))$.

However, with positive externalities, this equivalence between the two task orderings may not hold, thus creating the possibility of ordering reversal. Specifically, reversal occurs when $E\Pi(\boldsymbol{v}^*, (2, 1')) > E\Pi(\boldsymbol{v}^*, (1, 2'))$ and $\Delta_1 < E\Pi(\boldsymbol{v}^*, (2, 1')) - E\Pi(\boldsymbol{v}^*, (1, 2'))$; together, these conditions imply that $E\Pi(\boldsymbol{v}^*, (2, 1')) > E\Pi(\boldsymbol{v}^*, (1, \widehat{2'})) > E\Pi(\boldsymbol{v}^*, (1, 2'))$, hence the optimal ordering is $(2, 1')$ instead of $(1, \widehat{2'})$ (see Prop. 5(ii)).

Example 2. Consider a two-task project with the following parameters: $\alpha_1 = \alpha_2 = 0.4$, $\beta_1 = \beta_2 = 0.5$, $c = 1$ and $v = 75$. Both tasks then have the same threshold values $\bar{\gamma}_1 = \bar{\gamma}_2 = \frac{2}{3}$. Suppose further that $\gamma_1 = 0.5$ and $\gamma_2 = 0.80$. Then $\gamma_1 < \bar{\gamma}_1$ and $\gamma_2 > \bar{\gamma}_2$, so in the absence of positive externalities, $(1, \widehat{2}) \succ (2, 1)$.

Now let $\beta_1' = 0.9$, $\beta_2' = 0.6$. We see that $E\Pi(\boldsymbol{v}^*, (2, 1')) - E\Pi(\boldsymbol{v}^*, (1, 2')) = 42.225 - 34.875 = 7.35 > \Delta_1 = 0.625$. By Prop. 5(ii), the optimal ordering is $(2, 1')$. ‖

APPENDIX

Proof of Proposition 1: (i) Let $\gamma_1 \geq \bar{\gamma}_1$, $\gamma_2 \geq \bar{\gamma}_2$. In this case, $E\Pi(\nu^*, (i,j)) \geq E\Pi(\widetilde{\nu}^*, (i,j))$ for any (i,j). Moreover, we know that $E\Pi(\nu^*, (1,2)) = E\Pi(\nu^*, (2,1))$ (see Comment 1). Therefore, full contribution is optimal, and the Principal is indifferent between the two task orderings.

(ii) Let $\gamma_1 < \bar{\gamma}_1$, $\gamma_2 \geq \bar{\gamma}_2$. Suppose $i = 1$. Since $\gamma_1 < \bar{\gamma}_1$, $E\Pi(\widetilde{\nu}^*, (1,2)) > E\Pi(\nu^*, (1,2))$ (the Principal prefers not to incentivize effort in task 2). On the other hand, if $i = 2$, then $E\Pi(\widetilde{\nu}^*, (2,1)) \leq E\Pi(\nu^*, (2,1))$ (since $\gamma_2 \geq \bar{\gamma}_2$). To determine which task ordering yields more profit, recall that by Result 1), $E\Pi(\nu^*, (2,1)) = E\Pi(\nu^*, (1,2))$. Therefore, the condition $\gamma_1 < \bar{\gamma}_1$, which implies that $E\Pi(\widetilde{\nu}^*, (1,2)) > E\Pi(\nu^*, (1,2))$, likewise implies that $E\Pi(\widetilde{\nu}^*, (1,2)) > E\Pi(\nu^*, (2,1))$: the optimal ordering of tasks is $(1,2)$ and the optimal rewards $\widetilde{\nu}$.

(iii) The proof of part (ii) applies here with the task labels interchanged.

(iv) Let $\gamma_1 < \bar{\gamma}_1$, $\gamma_2 < \bar{\gamma}_2$. Suppose $i = 1$. Since $\gamma_1 < \bar{\gamma}_1$, $E\Pi(\widetilde{\nu}^*, (1,2)) > E\Pi(\nu^*, (1,2))$. On the other hand, if $i = 2$, then the $E\Pi(\widetilde{\nu}^*, (2,1)) > E\Pi(\nu^*, (2,1))$ (since $\gamma_2 < \bar{\gamma}_2$). Given $E\Pi(\nu^*, (2,1)) = E\Pi(\nu^*, (1,2))$, then

$$E\Pi(\widetilde{\nu}^*, (1,2)) > E\Pi(\widetilde{\nu}^*, (2,1)) \Leftrightarrow E\Pi(\widetilde{\nu}^*, (1,2)) - E\Pi(\nu^*, (1,2)) > E\Pi(\widetilde{\nu}^*, (2,1)) - E\Pi(\nu^*, (2,1)).$$

Using (13), we write the inequality above as

$$(1-\beta_1)\left[\beta_2\left(\tfrac{c}{\beta_2-\alpha_2}\right) - (\beta_2-\alpha_2)\gamma_1\nu\right] > (1-\beta_2)\left[\beta_1\left(\tfrac{c}{\beta_1-\alpha_1}\right) - (\beta_1-\alpha_1)\gamma_2\nu\right].$$

Simplifying, we get

$$(1-\beta_1)\left[\bar{\gamma}_1(\beta_2-\alpha_2)\nu - (\beta_2-\alpha_2)\gamma_1\nu\right] > (1-\beta_2)\left[\bar{\gamma}_2(\beta_1-\alpha_1)\nu - (\beta_1-\alpha_1)\gamma_2\nu\right]$$
$$(1-\beta_1)(\bar{\gamma}_1-\gamma_1)(\beta_2-\alpha_2)\nu > (1-\beta_2)(\bar{\gamma}_2-\gamma_2)(\beta_1-\alpha_1)\nu$$
$$\frac{\bar{\gamma}_1-\gamma_1}{\bar{\gamma}_2-\gamma_2} > \left(\frac{\beta_1-\alpha_1}{1-\beta_1}\right)\frac{1-\beta_2}{\beta_2-\alpha_2}. \tag{26}$$

This last inequality is condition (15).

By the same reasoning, $E\Pi(\widetilde{\nu}^*, (2,1)) > E\Pi(\widetilde{\nu}^*, (1,2))$ if and only if

$$\frac{\bar{\gamma}_2-\gamma_2}{\bar{\gamma}_1-\gamma_1} > \left(\frac{\beta_2-\alpha_2}{1-\beta_2}\right)\frac{1-\beta_1}{\beta_1-\alpha_1}.$$

This inequality is condition (16). ∎

Proof of Proposition 4: (i) If $\gamma_i \geq \bar{\gamma}_i$ for $i = 1,2$, then $(1,2') \succ (1,\widehat{2'})$ and $(2,1') \succ (2,\widehat{1'})$. If $E\Pi(\nu^*, (1,2')) > E\Pi(\nu^*, (2,1'))$, then $(1,2') \succ (2,1')$. If $E\Pi(\nu^*, (1,2')) = E\Pi(\nu^*, (2,1'))$, then $(1,2') \sim (2,1')$.

(ii) Suppose that $\gamma_1 < \bar{\gamma}_1$ and $\gamma_2 \geq \bar{\gamma}_2$. Then $(1,\widehat{2'}) \succ (1,2')$ and $(2,1') \succ (2,\widehat{1'})$. We see that

$$E\Pi(\nu^*, (1,\widehat{2'})) = E\Pi(\nu^*, (1,2')) + (1-\beta_1)(\beta_2-\alpha_2)\nu[\bar{\gamma}_1-\gamma_1]$$
$$> E\Pi(\nu^*, (2,1')),$$

since $E\Pi(\nu^*, (1,2')) \geq E\Pi(\nu^*, (2,1'))$ and $\gamma_1 < \bar{\gamma}_1$. Therefore, $(1,\widehat{2'}) \succ (2,1')$.

(iii) Suppose that $\gamma_1 \geq \bar{\gamma}_1$ and $\gamma_2 < \bar{\gamma}_2$. Then $(1, 2') \succ (1, \widehat{2'})$ and $(2, \widehat{1'}) \succ (2, 1')$. We see that

$$
\begin{aligned}
\mathrm{E\Pi}(\boldsymbol{v}^*, (2, \widehat{1'})) &= \mathrm{E\Pi}(\boldsymbol{v}^*, (2, 1')) + \Delta_2 \\
&= \mathrm{E\Pi}(\boldsymbol{v}^*, (2, 1')) + \Delta_2 - \mathrm{E\Pi}(\boldsymbol{v}^*, (1, 2')) + \mathrm{E\Pi}(\boldsymbol{v}^*, (1, 2')).
\end{aligned}
$$

Hence

$$
\mathrm{E\Pi}(\boldsymbol{v}^*, (2, \widehat{1'})) - \mathrm{E\Pi}(\boldsymbol{v}^*, (1, 2')) = \Delta_2 - \big[\mathrm{E\Pi}(\boldsymbol{v}^*, (1, 2')) - \mathrm{E\Pi}(\boldsymbol{v}^*, (2, 1')) \big].
$$

If the right-hand side is positive, then $\mathrm{E\Pi}(\boldsymbol{v}^*, (2, \widehat{1'})) > \mathrm{E\Pi}(\boldsymbol{v}^*, (1, 2'))$ and $(2, \widehat{1'}) \succ (1, 2')$. If the right-hand side is equal to zero, then $(2, \widehat{1'}) \sim (1, 2')$. Finally, if it is negative, then $(1, 2') \succ (2, \widehat{1'})$.

(iv) Suppose that $\gamma_i \geq \bar{\gamma}_i$ for $i = 1, 2$. Then $(1, \widehat{2'}) \succ (1, 2')$ and $(2, \widehat{1'}) \succ (2, 1')$. We see that

$$
\begin{aligned}
\mathrm{E\Pi}(\boldsymbol{v}^*, (2, \widehat{1'})) &= \mathrm{E\Pi}(\boldsymbol{v}^*, (2, 1')) + \Delta_2 \\
&= \mathrm{E\Pi}(\boldsymbol{v}^*, (2, 1')) + \Delta_2 - \mathrm{E\Pi}(\boldsymbol{v}^*, (1, \widehat{2'})) + \mathrm{E\Pi}(\boldsymbol{v}^*, (1, \widehat{2'})) \\
&= \mathrm{E\Pi}(\boldsymbol{v}^*, (2, 1')) + \Delta_2 - \mathrm{E\Pi}(\boldsymbol{v}^*, (1, 2')) - \Delta_1 + \mathrm{E\Pi}(\boldsymbol{v}^*, (1, \widehat{2'})).
\end{aligned}
$$

Hence

$$
\mathrm{E\Pi}(\boldsymbol{v}^*, (2, \widehat{1'})) - \mathrm{E\Pi}(\boldsymbol{v}^*, (1, \widehat{2'})) = \big[\Delta_2 - \Delta_1 \big] - \big[\mathrm{E\Pi}(\boldsymbol{v}^*, (1, 2')) - \mathrm{E\Pi}(\boldsymbol{v}^*, (2, 1')) \big].
$$

If the right-hand side is positive, then $\mathrm{E\Pi}(\boldsymbol{v}^*, (2, \widehat{1'})) > \mathrm{E\Pi}(\boldsymbol{v}^*, (1, \widehat{2'}))$ and $(2, \widehat{1'}) \succ (1, \widehat{2'})$. If the right-hand side is equal to zero, then $(2, \widehat{1'}) \sim (1, \widehat{2'})$. Finally, if it is negative, then $(1, \widehat{2'}) \succ (2, \widehat{1'})$. ∎

Proof of Proposition 5: The reasoning is analogous to the proof of Proposition 4, with the task labels interchanged. ∎

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