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OPTIMAL RETURN IN A MODEL OF BANK SMALL-BUSINESS FINANCING

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Abstract

This paper develops a simple model showing how banks can increase the access to finance of small, risky firms by mitigating coordination problems among investors. If investors observe a biased signal about the true implementation cost of real sector projects, the model can be solved for a switching equilibrium in the classical global games approach. We show that the socially optimal interest rate that maximizes the probability of success of the firm is higher than the risk-free rate. Yet if banks maximize investors' expected return, they would choose an interest higher than the socially optimal one. This gives rise to a form of credit rationing, which stems from the funding constraints of the banks.

Keywords: Bank finance, small business, global games, switching equilibrium, optimal return.
JEL Classification index: D82, C72, G21, G32.

1 Introduction

In modern market economies, small businesses are an essential vector of growth, job-creation and innovation. In contrast to large firms, small firms are more "informationally opaque" insofar as they do not disclose certified risk rates or credit scores and do not publish systematically audited financial statements or data on collateral (Berger and Udell, 2006). As a consequence, access to external capital is a relevant constraint to the growth of small, entrepreneurial

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firms (Beck and Demirguc-Kunt, 2006). According to a large consensus in the economic literature, bank relationship lending has emerged as an efficient way of overcoming these informational frictions characterizing small firms (Petersen and Rajan, 1994). Unsurprisingly, small and medium enterprises (SMEs) are largely dependent on banks for their external finance. For example, taking stock from survey data on Western European countries, Berger and Schaeck (2011) find that 60% of the SMEs surveyed rely on bank-financing, with most funds coming from small, regional financial institutions.[1]

Banks are key intermediaries between small firms and investors. A large body of literature has focused on the role of banks in facilitating the access to credit of small borrowers by engaging in relationship lending (see Boot, 2000, for an overview). Through this strong firm-creditor relationship, the bank can acquire specific information about the firm, such as the entrepreneur's talent, future prospects and business environment, which generally leads to an increased availability of credit to small firms (Petersen and Rajan, 1994; Berger and Udell, 1995; Cole, 1998; Bartoli, Ferri, Murro and Rotondi, 2013). The effect of this type of lending on the cost of capital is, however, less clear. From a theoretical standpoint, a closer bank-firm relationship should reduce the bank's costs of acquiring information, and thus reduce the loan risk premium. If competition in the banking sector is strong, these cost savings should be passed on to borrowers in the form of lower interest rates (Petersen and Rajan, 1994; Boot and Thakor, 1994). However, if the "soft" information specific to bank lending relationships cannot be easily accessed by external investors, then the bank can acquire some form of "informational market power" that might justify a higher interest rate (as in Greenbaum, Kanatas and Venezia, 1989; Rajan, 1992). The empirical literature has not yet reached a definitive conclusion on this important issue.[2]

A related literature has focused on the bank-investor relationship. In general, smaller-sized, local banks are more likely to lend to small, less transparent firms (Berger and Udell, 2002; Scott, 2004; Berger and Black, 2011). At the same time, these banks are more likely to face difficulties when raising capital themselves, because investors could perceive their loan portfolios as bearing a high risk. Indeed, recent empirical evidence suggests that small, local banks

[1] They also find that half of the firms that rely on bank funding obtain it from a single bank.

[2] Looking at US data, Berger and Udell (1995) find that strong bank-borrower relationships are empirically associated with lower loan interest rates, whereas Petersen and Rajan (1994) find no statistically significant link. Taking evidence from European data, Degryse and Van Cayseele (2000) show that loan rates actually increase in the duration of the relationship, whereas Harhoff and Körting (1998) find no effect.

are constrained in their access to external finance (Paravisini, 2008; Banerjee and Duflo, 2008; Khwaja and Mian, 2008; Iyer, Peydro, da Rocha-Lopes and Schoar, 2014).

A complete analysis of small-business bank finance must aggregate the two sides of the market, with banks playing a pivotal role between firms and investors. This paper aims to do so by developing a model of small-business finance that takes into account both a technological risk specific to small, entrepreneurial firms and a particular financial risk stemming from an inability of bank creditors to coordinate their investment decisions. We emphasize the importance of funding constraints for banks themselves and how these can affect the availability and cost of capital for small entrepreneurial firms. Our model makes a simple but important point. We show that the "socially optimal" interest rates on loans are higher than one might expect under a perfectly competitive banking sector because higher interest rates mitigate the coordination problem arising among investors. However, in a decentralized organization of the bank intermediation market, interest rates would be higher than the socially optimal, thus bringing about a specific form of market inefficiency.

In our model, banks play a key function of acquiring information about investment opportunities and facilitating the access to finance of small firms that cannot raise funds in capital markets. We assume that the financial sector is perfectly competitive, such that banks earn zero profits. This competitive environment will drive banks to maximize the expected return of the risk neutral investors who fund the bank (Diamond and Dybvig, 1983; Rochet and Vives, 2004).[3] Investors are the only agents endowed with wealth and they face a choice between investing in a risk-free asset which yields a safe return r, or placing their funds with the bank that will invest them in the firm, for a risky return, R.

The real sector is represented by a small entrepreneurial firm that owns an innovative, but risky technology and seeks to raise capital to implement it. The firm's output is increasing in the amount of capital raised according to a linear technology. Technological risk is captured by a stochastic cost to be realized during the implementation of the project. Because the firm cannot access capital markets directly, it will engage in a lending relationship with the bank, allowing the latter to observe the true distribution of the technological shock. The bank will then share this information with the investors whose funds

[3] Allowing for risk-averse investors would not change the main insights of the model (the main implication would be stronger).

it seeks to attract.[4]

Given the linear technology in capital, the probability that the firm succeeds is increasing in the availability of capital. At the same time, the amount of available capital depends on the number of investors who place their funds with the bank. This brings about a typical coordination problem, which has be shown to generate multiple equilibria under perfect information (Diamond and Dybvig, 1983).[5] Of course, the assumption of perfect information is too strong, particularly when applied to highly innovative entrepreneurial firms. If we assume instead that investors observe only a noisy signal about the true technological cost, the problem can be analyzed as a standard "global game" (Carlsson and Van Damme, 1993). Morris and Shin (1998) have proposed an elegant equilibrium concept that applies to $n-$player coordination games with noisy information.[6] In this methodology, the "fundamentals of the economy" are captured by a state variable related to the ability of the economy to cope with external shocks. Under ex-post and ex-ante uncertainty about these fundamentals, the multiple equilibria feature of the perfect-information model vanishes, and the coordination problem is characterized by a unique "threshold or switching" equilibrium. Our solution builds on this established methodology.[7] We show that the investors' coordination problem presents a switching equilibrium, where the project succeeds if the technological cost is below a critical value, and fails in the opposite case.

The original contribution of this paper is to show that this critical threshold depends in a nonlinear way on the return, R, chosen by the bank. The impact of the return on capital on the critical cost is driven by two opposite effects. On the one hand, higher returns make the decision to lend to the bank more appealing and support the collective decision to invest, which in turn increases the probability of success for the firm. On the other hand, higher interest rates raise the capital cost of the firm and have an adverse effect on its probability

[4]We thus implicitly assume that private investors are not able to gauge the technological risk on their own.

[5]If the technological cost is not too high, there is a high-risk, Pareto-dominant equilibrium, characterized by all investors investing regardless to the return set by the bank ($R > r$); whatever the positive cost, there is also a zero-risk Pareto-dominated equilibrium in which no investor participates and the project fails.

[6]The methodology has been applied to study various economic problems involving coordination frictions, such as currency crises, bank runs, credit risk and illiquidity debt default (Morris and Shin, 2001; Morris and Shin, 2003; Morris and Shin, 2004; Morris and Shin, 2009; Bebchuk and Goldstein, 2011).

[7]While in existing global games models the state variable has a rather "generic" nature, in this paper, the state variable can be related directly to the technological uncertainty specific to new investment projects.

of success. We show that there exists a "socially optimal interest rate" that yields the highest chances of success to the entrepreneurial firm. This interest rate is higher than the risk-free interest rate and this "abnormal margin" stems from the coordination problems among investors. Moreover, in a decentralized organization of the banking sector, banks would chose an interest rate higher than the social optimal level, thus generating a form of market inefficiency. The bank's funding constraint, which is the result of coordination failures among investors, is at the origin of this inefficiency.

Our model is related to several stands of literature. First, our work contributes to a large literature on applied global games by modeling an inefficient allocation of capital to the real sector as a result of coordination problems among bank creditors. In a framework related to ours, Bebchuk and Goldstein (2011) model a credit-market freeze which arises from a coordination failure among banks to extend credit to operating firms. In their model, real sector projects are highly interdependent such that if all banks provide credit, all firms do well and banks can recover their investment. Yet if several banks refuse to lend, then all firms collapse and it is rational for all banks to refuse loans. They study how policy responses can get an economy out of credit-market freezes. In our model, banks can mitigate coordination failures among investors through higher interest rates. These higher interest rates, however, bring about an inefficient shortage of credit in the economy: more capital could have been directed towards real sector projects in the absence of such coordination problems. Generally, theories on credit rationing inefficiencies focus on borrower adverse selection problems as in the seminal paper by Stiglitz and Weiss (1981). Here we show that bank fundings constraints can generate similar inefficiencies. The same intuition is present in Agur (2012; 2013) who shows how credit rationing can arise when banks are funding constrained.

The paper is organized as follows. The next section lays out the main assumptions. Section 3 presents the equilibrium solution. The question of how banks determine the optimal interest rate is addressed in Section 4. The last section concludes.

2 Main assumptions

The model is cast as a game between a continuum of small investors, a bank operating in a competitive environment and a small entrepreneurial firm. The firm has no endowment and needs to borrow to invest in a new project. Investors are the only agents endowed with wealth but face extreme information asymmetry if they lend directly to the firm. The bank can reduce this asymmetry by engaging in relationship lending with the firm.

A. The firm

The firm is the owner of a new technology or new productive process that requires capital in order to be implemented. We assume that the firm has no funds and needs to borrow to finance the project. The firm's output is a linear function in the capital used in the production process: $Y = (1 + A)K$, with K being the amount of capital and A a positive parameter characteristic of the marginal product of capital $(1 + A)$. Implementation of the project is subject to substantial technological uncertainty. To keep the model as simple as possible, we model this uncertainty as a stochastic technological cost $\tilde{c}$. For instance, for a given project, the number of design hours or research time can exceed by far the normal value (i.e., the long run average). For analytical convenience, we assume that this shock follows a normal distribution $\tilde{c} \rightsquigarrow N(\bar{c}, \sigma^2)$, with a mean value $\bar{c} > 0$ and a precision $\alpha = 1/\sigma^2$.[8]

Given these assumptions the profit function can be written as:

$$\tilde{\pi} = (1 + A)K - (1 + R)K - \tilde{c} \tag{1}$$

where R is the return required by the bank for the loan, with $R \leq A$.

B. The Bank

The bank operates in a competitive environment, making zero profits.[9] It channels funds from investors to entrepreneurs. In this intermediation process, the bank serves two important functions.

First, by engaging in a lending relationship with the firm, the bank can obtain private information about the worthiness of the project. More precisely, we assume that the bank, by acquiring knowledge about the firm's business environment or the entrepreneur, is able to observe the true distribution of technological shocks; it will then share this information with (ex-ante uninformed)

[8]Normally the cost cannot be negative. Hence the mean should be large enough and the variance small enough such that $\Pr[c < 0]$ is negligible.

[9]Despite the competitive nature of the banking sector, the firm only borrows from one bank, which reduces our model to a "one bank" model. This assumption is strongly supported by the empirical findings which show how small firms generally borrow from a single, most of the time local, bank (see, for example, Agarwal and Hauswald, 2010; Berger and Schaeck, 2011).

investors. Investors who want to seize this opportunity become bank's clients, i.e., should they decide to participate to the project, they commit themselves to do it through the intermediation of the bank.

Secondly, the bank pools resources from many small investors who decide to participate to the risky collective project and invests them in the firm. Thus the simplified balance sheet of the bank will record investors' deposits as liabilities and a "massive" loan to the firm as the main asset. Since we consider that banks operate in a competitive environment, profits should tend to zero; as a consequence, banks apply the same interest rate R on both the loan to firms and deposits. This competitive environment drives the bank to maximize investors' expected return.

C. Investors

There is a continuum of $N = 1$ risk-neutral investors, each endowed with one unit of wealth. They have the choice between placing their funds in the banking sector or investing them in a safe asset which yields a return r. The return promised by the bank is risky and depends on the success of the entrepreneurial firm. If the investment in the firm proves to be successful, investors receive the return R, with $R \geq r > 0$. If the firm fails, investors recover a liquidation value, $v < 1$. We denote the proportion of investors who lend to the bank by ℓ, with $\ell \in [0, 1]$.

At the outset of the game investors are "uninformed", they have no means of inferring the worthiness of the new project from public information. By entering in a contract with the bank, the latter shares with them information about the true distribution of the technological shock. At that moment the distribution of shocks $\tilde{c} \rightsquigarrow N(\bar{c}, \sigma^2)$ becomes common knowledge.

Once that the shock (technological cost) is realized, investors receive a signal about the true value of c. More precisely, an investor $i \in (0, 1)$ will observe

$$x_i = c + \epsilon_i \tag{2}$$

where $\epsilon_i \rightsquigarrow N(0, \tau^2)$. The precision of the signal is denoted by $\beta = 1/\tau^2$.

Investors' payoff, contingent upon their individual and joint decisions, as well as the realization of the shock c, can be written as:

$$U = \begin{cases} \text{lend to the bank} & \begin{cases} 1+R & \text{if } \pi \geq 0 \\ v & \text{if } \pi < 0 \end{cases} \\ \text{invest in safe asset} & 1+r \end{cases}$$

D. Timing

The sequence of decisions and the information structure of the game are depicted in Figure 1.

t=0	t=1	t=2
• The firm and the bank enter the credit relationship • Bank decides on R	• Technological shock is realized • Investors observe a noisy signal about the technological shock • Investors decide on lending to the bank or investing in the safe asset • The bank transfers capital to the firm	• Investors receive R if project is successful, and v otherwise

Figure 1: Timing

At time $t = 0$, the firm and the bank enter the credit relationship. The bank observes the distribution of technological shocks and shares the information with investors. It sets the interest rate R.

At time $t = 1$, the technological shock is realized. Investors observe a noisy signal. They decide whether to lend their funds to the bank, based on the signals they receive and the return R promised by the bank in case of success of the project. The supply of funds for the project is thus equal to the number of investors that choose the participation strategy, ℓ. The bank transfers the funds to the firm.

At time $t = 2$, the firm makes a positive profit or not (is bankrupt). If the profit is positive, the firm pays to investors the contracted return R; if not, the firm is liquidated and investors get the residual value v.

3 The Equilibrium

Given the firm's production technology in Equation (1), its demand for funds is infinitely elastic. Thus the equilibrium capital is given by the supply of funds: $K = \ell$ and the firm's profit function can be re-written as:

$$\tilde{\pi} = (A - R)\ell - \tilde{c}. \tag{3}$$

Equation 3 points out the importance of coordination among investors. When the cost is zero (or below), the project succeeds even if only one investor has participated. When $c > (A - R)$, the firm fails even if all investors ($\ell = 1$) have participated. When c lies on the interval $(0, A - R)$, the firm succeeds only if a critical mass of investors participate; but the decision to participate depends on every investor's belief about the beliefs of the others.

The signals x_i that investors receive convey information not only about the technological cost c, but also about the signals that other investors receive. At the extreme, when $c = 0$, an investor should lend no matter what the others do. Consider now an investor receiving a signal sightly above zero. This investor infers that other investors might have received signals equal to or below zero, and thus have a dominant strategy to lend. Then it is also optimal for him to lend as well. Applying the same logic several times, we can establish a boundary well above zero below which investors should lend. At the same time, investors have a dominant action not to lend when $c > (A - R)$, because projects fail even if every investor participates. So when an investor receives a signal slightly below $(A - R)$ he is pessimistic about the probability of success of the firm and prefers not to lend. Again, we can apply a backward reasoning and establish a boundary well below $(A - R)$ above which investors do not lend. A formal proof, presented in Morris and Shin (1998; 2004), shows that these two boundaries coincide, such that the coordination problem admits a unique equilibrium characterized by a "switching strategy" (invest / do not invest) around a critical signal. For brevity, we do not repeat here their argument of the proof.

The equilibrium of the game is thus characterized by two thresholds, a "critical signal" x^* driving investors' decision (invest / do not invest), and a "critical cost" c^* for which the firm's project is right on the edge between failing or not. Proposition 1 states our basic equilibrium result.

PROPOSITION 1. The problem presents a single equilibrium provided that the precision of the signal is large enough, more precisely if the sufficient (not necessary) condition $\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{A-R}$ holds. The equilibrium critical cost, below which the firm's project succeeds is implicitly defined by equation:

$$c^* = (A-R)\Phi\left(\frac{\alpha}{\sqrt{\beta}}\left[c^* - \bar{c} - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right)\right]\right). \tag{4}$$

PROOF. Following standard resolution steps (see Morris and Shin, 2004), the two thresholds, x^* and c^* can be determined as the solution to a system of two equations.

First, when Nature draws a cost c, the *proportion of investors who lend* is equal to the frequency of investors who receive a signal below the *critical signal* x^*:

$$\ell = \Pr(x_i < x^*|c) = \Pr(\epsilon_i < x^* - c) = \Phi(\sqrt{\beta}(x^* - c)), \tag{5}$$

where $\Phi()$ is the c.d.f. of the standard normal distribution.

Turning to the *critical cost* c^*, such as for $c > c^*$ the firm fails and for $c \leq c^*$ the firm succeeds. Given the profit function in Equation (3), c^* can be written: $c^* = (A-R)\ell$. Moreover, based on Equation (5) above, the proportion of investors who lend when the cost is exactly c^* is $\Phi(\sqrt{\beta}(x^* - c^*))$. It follows that the critical cost c^* is implicitly defined by:

$$c^* = (A-R)\Phi(\sqrt{\beta}(x^* - c^*)). \tag{6}$$

This gives us the first equation in c^* and x^*.

Second, given our assumptions about the normality of the distributions of costs and signals, when an investor i receives a signal x_i, his posterior distribution of c is also normal with mean $\left(\frac{\alpha\bar{c} + \beta x_i}{\alpha+\beta}\right)$ and precision $(\alpha+\beta)$. So, for an investor with signal x_i, the probability of failure of the firm is:

$$\Pr(c > c^*|x_i) = 1 - \Phi\left(\sqrt{\alpha+\beta}\left[c^* - \frac{\alpha\bar{c} + \beta x_i}{\alpha+\beta}\right]\right). \tag{7}$$

Among the continuum of investors, there exists one who receives exactly the critical signal x^*; this individual is indifferent between lending or not to the bank. His indifference condition can be written as:

$$(1+R)\left[1 - \Pr(c > c^*|x^*)\right] + v\Pr(c > c^*|x^*) = (1+r), \tag{8}$$

which is equivalent to:

$$\Phi\left(\sqrt{\alpha+\beta}\left[c^* - \frac{\alpha\bar{c}+\beta x^*}{\alpha+\beta}\right]\right) = \frac{1+r-v}{1+R-v}. \tag{9}$$

After some calculations it follows that:

$$x^* - c^* = \frac{\alpha}{\beta}(c^* - \bar{c}) - \frac{\sqrt{\alpha+\beta}}{\beta}\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right). \tag{10}$$

which gives us the second equation in x^* and c^*.

Our *equilibrium* critical thresholds x^*_E and c^*_E are thus the solution to the system of equations (6) and (10). By substituting $(x^* - c^*)$ as given by (10) in (6), we get that c^*_E is the solution to the equation:

$$c^* = (A-R)\Phi\left(\frac{\alpha}{\sqrt{\beta}}\left[c^* - \bar{c} - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right)\right]\right).$$

Graphically, the equilibrium critical cost (the failure threshold) c^*_E is obtained at the intersection between the 45° line and the scaled-up cumulative normal distribution with mean $\left[\bar{c} + \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right)\right]$ and standard deviation $\frac{\alpha}{\sqrt{\beta}}$. A single solution is guaranteed when the slope of the right hand-side of Equation (4) less than one, that is: $(A-R)\phi(\cdot)\frac{\alpha}{\sqrt{\beta}} < 1$, where $\phi(\cdot)$ is the p.d.f. of the standard normal distribution. Given that $\phi(\cdot) < \frac{1}{\sqrt{2\pi}}$, then the unique solution exists if the sufficient (not necessary) condition $\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{A-R}$ holds. *Q.E.D.*

The equilibrium critical cost c^*_E is represented at point E in Figure 2, draw for parameters that fulfill the single equilibrium condition.[10] Notice that the actual proportion of investors who lend to the bank is given by those investors who receive a signal below this critical signal. Since $x_i = c + \epsilon_i$, the proportion of investors who place their funds in the bank is a function of the realized shock: i.e. $\ell(c) = \Pr[x_i < x^*(c^*)|c] = \Phi(\sqrt{\beta}(x^*(c^*) - c))$. Figure 3 illustrates the proportion of investors who lend as a function of the actual realization of the cost c considering the same parameters as for the former figure.

Clearly, there exists a range of costs for which viable projects fail due to lack

[10]More precisely $c \sim N(0.5, \frac{1}{10})$, $\epsilon \sim N(0, \frac{1}{1000})$, $r = 0$, $A = 2$, $v = 0.1$, $R = 0.71$.

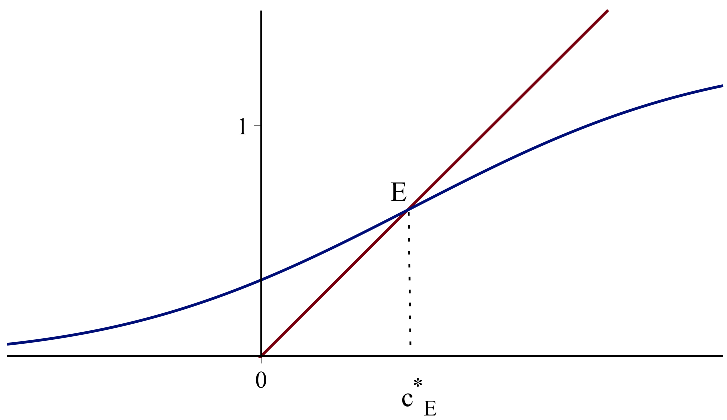

Figure 2: The equilibrium critical cost

of investor coordination. In particular, for $c \in (c_E^*, A - R)$, the firm's project would succeed if all investors coordinated and lend to the bank (but they do not). This gives rise to a form of "inefficient credit freeze" which is the outcome of investors' failure to coordinate on an otherwise "good" project.

The uniqueness of equilibrium allows us to analyze in a straightforward way how the equilibrium critical cost c_E^* responds to changes in the parameters of our model. At the same time, since the probability of success of the project, $\Pr[c < c_E^*] = \Phi\left(\sqrt{\alpha}\,(c^* - \bar{c})\right)$, is monotonously increasing in c_E^*, these comparative statics allow us to understand how this probability is impacted by the model parameters. We present all comparative statics in Appendix A. First, we show that a lower risk-free rate, r, prompts more investors to participate, and thus raises the probability of success of the firm. This might correspond to a situation where the key interest rate of monetary policy is cut in response to adverse macroeconomic shocks; in our model, this would induce more bank lending to the real sector. A similar effect is brought about by a smaller average technological cost faced by the firm, $\bar{c}$. This would happen in a period of extreme innovation (such as the IT revolution in the late nineties). A closer lending relationship, which translates to the ability of the bank to secure a higher liquidation value, v, also increases the firm's access to finance.[11] Notice that a very

[11] Governmental guarantee programs, such as the US Small Business Administration 7(a) Loan Program, which guarantees bank loans to small businesses, should have a similar impact on the liquidation value of the project, and would result in an increased access to finance.

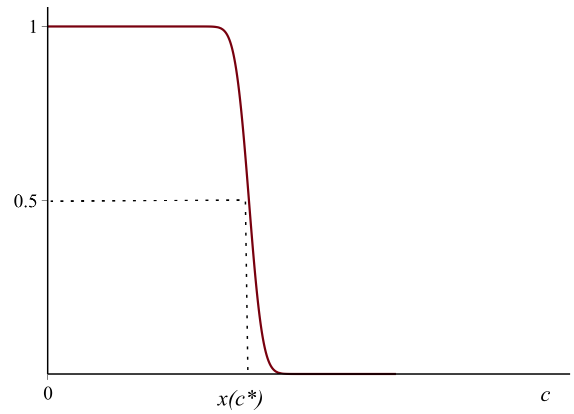

Figure 3: Proportion $l(c)$ of investors who lend to the bank

favorable macroeconomic environment might have an ambiguous effect on the success probability; indeed, in such a context, the average cost $\bar{c}$ might decline, while at the same time the central bank would push up the short-term interest rate r.

Finally, a key determinant of the equilibrium critical cost is the return R chosen by the bank. In the next section, we study more in depth this relationship.

4 Optimal return and market efficiency

4.1 The relationship between the return and the equilibrium critical cost

Intuitively, the relationship between c^* (and hence the probability of success of the firm) and R is driven by two opposite effects.[12] On the one hand, a higher return R should increase the participation rate ℓ in the risky project and the chances that the project succeeds. On the other hand, a higher R raises the cost of capital, so it increases the probability that the firm will default on its liabilities. We can state:

[12]From now on we refer only to the equilibrium critical cost and signal. To avoid excessively complex notation, we can drop the subscript E.

PROPOSITION 2. The equilibrium critical cost $c^*(R)$ admits at least one maximum for $\hat{R} \in (r, A)$.

PROOF. From Equation (4) we know that on the interval $R \in [r, A]$, the function $c^*(R)$ is continuous, positive and lower than $(A - R)$. Moreover, at the extremes we have $\lim_{R \to r} c^* = 0$ and $\lim_{R \to A} c^* = 0$[13] Applying the implicit function theorem (see Appendix) we get:

$$\frac{dc^*}{dR} = \frac{\sqrt{\frac{\alpha+\beta}{\beta}} \frac{(1+r-v)(A-R)}{(1+R-v)^2} \frac{\phi(\cdot)}{\phi\left(\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right)\right)} - \Phi(\cdot)}{1 - (A-R)\frac{\alpha}{\sqrt{\beta}}\phi(\cdot)}, \tag{11}$$

where $\Phi(\cdot) = \Phi\left(\frac{\alpha}{\sqrt{\beta}}\left[c^* - \bar{c} - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right)\right]\right)$ and
$\phi(\cdot) = \phi\left(\frac{\alpha}{\sqrt{\beta}}\left[c^* - \bar{c} - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right)\right]\right)$.

In Equation (11), we can check that $\left[\frac{dc^*}{dR}\right]_{R=A} < 0$. Indeed, the denominator of expression (11) is always positive because $1 - (A-R)\frac{\alpha}{\sqrt{\beta}}\phi(\cdot) \geq 1 - (A-R)\frac{\alpha}{\sqrt{\beta}}\frac{1}{\sqrt{2\pi}} > 0$, given the uniqueness condition $\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{A-R}$. The sign of derivative is thus the sign of $-\Phi(\cdot)$; for $R = A$, we have $\Phi(\cdot) > 0$. This suffices to prove that $c^*(R)$ admits at least one maximum for $\hat{R} \in (r, A)$. Moreover, because $\lim_{R \to r} c^* = 0$, this maximum is necessarily higher than the risk-free rate r. *Q.E.D.*

To bring additional intuition for this result, Figure 4 uses a numerical simulation to represent the critical cost c^* as a function of R. Parameter values are $c \sim N(0.5, \frac{1}{10})$, $\epsilon \sim N(0, \frac{1}{1000})$, $r = 0$, $A = 2$ and $v = 0.1$. The function $c^*(R)$ has a reverse U-shape.[14] The highest c^* is obtained for $\hat{R} = 0.71$. For this interest rate, the equilibrium critical cost below which the firm succeeds is $c^*(0.71) = 0.581$.

Our main result becomes even sharper in a special case of the game in which private signals become infinitely precise, i.e. $\beta \to \infty$. Under this assumption the condition for uniqueness always holds and x_E^* and c_E^* will converge to the

[13] Since $\Phi^{-1}(1) = \infty$ and $\Phi(-\infty) = 0$.

[14] This shape is obtained for a very wide range of parameters (that fulfill the uniqueness of equilibrium condition).

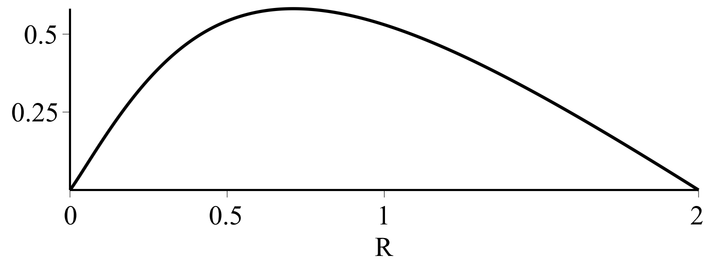

Figure 4: Evolution of threshold equilibrium c^*_E as a function of R

same value:

$$c^*_{\beta\to\infty} = x^*_{\beta\to\infty} = (A-R)\Phi\left(-\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right)\right) = \frac{(A-R)\,(R-r)}{1+R-v}. \quad (12)$$

We can check that the shape of the function $c^*_{\beta\to\infty}(R)$ matches the form presented in Figure 4. Indeed, $c^*_{\beta\to\infty}(r) = c^*_{\beta\to\infty}(A) = 0$. From the derivative

$$\frac{dc^*_{\beta\to\infty}}{dR} = \frac{(1+r-v)\,(1+A-v)}{(1+R-v)^2} - 1.$$

we can infer that the function is concave, with $\frac{d(c^*_{\beta\to\infty})^2}{d^2R} < 0$. The maximum critical cost is obtained for a $\hat{R}$ implicitly defined by:

$$1+\hat{R}-v = \sqrt{(1+r-v)(1+A-v)}$$

Furthermore, for small values of $(\hat{R}-v)$, $(r-v)$ and $(A-v)$ a log approximation yields: $\hat{R} \approx 0.5(r+A)$. Interestingly, $\partial\hat{R}/\partial r = 0.5$; the residual value v has only a second order effect that can be neglected.

4.2 Social vs. decentralized optimal return

In this section we compare the optimal return chosen by the bank with a "socially optimal one". We define this later return as the return a chosen by a "benevolent social planner" aiming to maximize employment and the overall economic performance of the start-up sector. It follows that the optimal interest rate maximizes the chance o survival of startup firms. Since the probability of success of a project $\Pr[c < c^*] = \Phi\left(\sqrt{\alpha}\left(c^* - \bar{c}\right)\right)$ is monotonously increasing in c^*, the maximum of this probability is obtained for the maximum of c^*. Thus the return $\hat{R}$ which maximizes c^* (as defined above) can be interpreted as this socially optimal return.

In a decentralized economy, we have argued that banks aim to maximize the expected return of their investors. Thus, the bank's optimization problem can be written as:

$$\max_R \left\{\Pr[c < c^*](1 + R) + (1 - \Pr[c < c^*])v\right\}.$$

PROPOSITION 3. The optimal interest rate that maximizes the expected return of investors, $\tilde{R}$, is higher than the return that maximizes the probability of success of the firm, $\hat{R}$.

PROOF. By denoting $\Pr[c < c^*] = \Phi\left(\sqrt{\alpha}\left(c^* - \bar{c}\right)\right)$ and $\Pr[c = c^*] = \varphi\left(\sqrt{\alpha}\left(c^* - \bar{c}\right)\right)$, the first order condition can be written as:

$$\frac{dc^*}{dR} = -\frac{\Phi(.)}{\varphi(.)}(1 + R - v)^{-1} < 0. \tag{13}$$

In the proof of Proposition 2, we have shown that there exists a return $\hat{R}$ that maximizes c^* and thus $\Pr[c < c^*]$, i.e. $\left[\frac{dc^*}{dR}\right]_{R=\hat{R}} = 0$. From condition (13) it follows that the optimal interest rate $\tilde{R}$ that maximizes the expected return of investors is reached on the downward slope of the curve $c^*(R)$ (since for the optimum return $\tilde{R}$ the derivative dc^*/dR takes a negative value). Thus this interest rate is necessarily higher than the one that maximizes the firm's chances of success ($\tilde{R} > \hat{R}$) and, as a result, higher than the risk-free rate. *QED*.

Figure 5 represents the investors' expected return (the full line) and the probability of success of the firm $\Pr[c < c^*]$ (the dashed line) a function of R, using the same parameter values as in Figure 4. The maximum probability of success of the firm of 60% is obtained for the return $\hat{R} = 0.71$ (that maximizes the critical cost). The return that maximizes investors' expected return is $\tilde{R} = 0.92$ ($\tilde{R} > \hat{R}$). For this decentralized optimal return, the probability of success of the firm falls to 57%.

We can summarize the main findings of this section as follows:

a. If the coordination risk can be ruled out, for instance if the bank raises funds from a single large investor (instead of many small investors), the chance

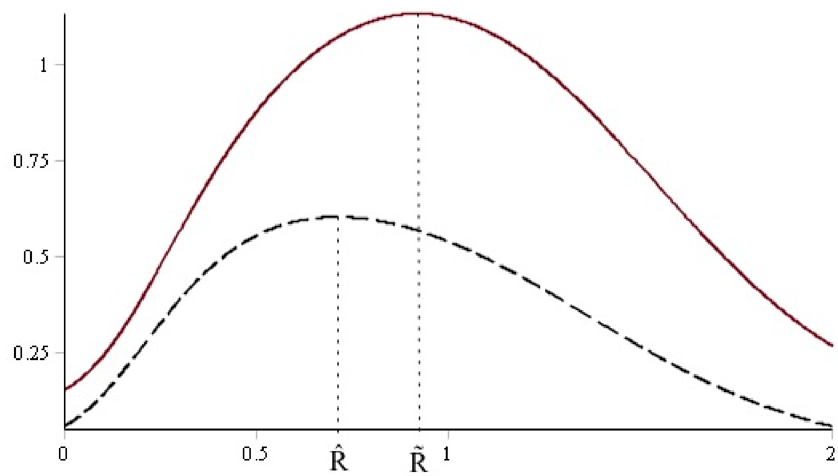

Figure 5: Firm's probability of success and investors' expected return

that the firm succeeds (makes a positive profit) is given by the probability $\Pr[c < (A - R)]$ (given Equation 3). Obviously, a competitive bank would maximize these chances if it sets $R = r$.

b. In the presence of coordination risk, the socially optimal interest rate that maximizes the probability of success of the firm is higher than the risk-free rate. Such high interest rates are not the consequence of banks abusing of some form of market power; to the contrary, the high rate is beneficial to the firm, insofar as it helps relaxing the funding constraint.

c. In a decentralized environment, banks aiming to maximize investors return would however choose an interest rate even higher than the socially optimal rate, thus pushing down the probability of success of the firm. Such form of inefficiency is built in the decentralized organization of the bank intermediation market.[15]

[15] In this model we assumed that investors are risk neutral. Should we instead have assumed that investors are risk-averse, then the appropriate objective of the bank would be to maximize expected utility. In this case, the optimal interest rate that maximize expected utility should be even higher than the interest rate that maximize expected returns.

5 Conclusion

In a world where small enterprises are a fundamental engine of growth, understanding how banks can facilitate their access to external funds and contribute to their development is an important issue. This paper has developed a simple model of bank lending that highlights how banks' funding constraints impact the cost and availability of external finance for small entrepreneurial firms. In our model, the bank performs two important missions: it collects and shares specific information about the worthiness of a new project, and it facilitate coordination among investors around that project. We have shown that the possibility that investors fail to coordinate brings about a strategic risk that compounds its effect to the intrinsic technological risk of the project.

We analyze the equilibrium behavior of investors in a standard global games approach. We show that there exists an equilibrium critical cost which separates the failure and success states of the project. This cost depends in a non-linear way on the return to capital chosen by the bank. The return on capital has two opposite effects on the firm's chances of success. On the one hand, a higher return mitigates the coordination problem by incentivizing investors to participate to the collective project thereby increasing the volume of available capital. On the other hand, a higher return raises the overall cost of capital, which increases the probability that the firm will default. There is a clear trade-off between the price of capital and the availability of funds. The original measure of availability developed in this paper reflects the complex decision of investors who take into account both the technological risk and the financial-strategic uncertainty.

We show that a social planner aiming to maximize the overall performance of the economy (employment) should not bid interest rates down to the risk-free rate. Instead, there exists an optimal interest rate which maximizes the probability of success of the firm and this interest rate is higher than the risk-free interest rate. Interestingly, this finding stems from a model featuring a competitive banking sector. Unlike other papers in which the wedge between the risk-free interest rate and the project loan rate is signaling an abuse of market power (Rajan, 1992; Petersen and Rajan, 1995), our analysis shows that a high loan rate can be in the interest of the firm, as a mean to relax the funding constraint.

However, if in a decentralized economy banks aim at maximizing investors' expected return, they would set the interest rate at a level higher than the rate chosen by the social planner. This entails a specific form of allocative inefficiency that cannot be ruled out spontaneously. Hence our analysis also points out a limit of the decentralized organization of the banking intermediation: the optimal interest rate from investors' point of view might not match the socially optimal rate that fully supports job creation and real sector projects' survival.

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A Appendix A: Comparative statics on the critical cost c^*

In this Appendix we analyze how the equilibrium critical cost c^* varies when R, $r, \bar{c}$ and v change. An increase in c^* is tantamount to an increase in chances that the project succeeds. Start by defining the function:

$$I(c^*, R, r, \bar{c}, v) = c^* - (A-R)\Phi\left(\frac{\alpha}{\sqrt{\beta}}\left[c^* - \bar{c} - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right)\right]\right) = 0$$

We have:

$$\frac{\partial I}{\partial c^*} = 1 - (A-R)\frac{\alpha}{\sqrt{\beta}}\phi(\cdot) \geq 1 - (A-R)\frac{\alpha}{\sqrt{\beta}}\frac{1}{\sqrt{2\pi}} > 0,$$

given the imposed condition for equilibrium uniqueness $\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{A-R}$, and

$$\begin{aligned}
\frac{\partial I}{\partial R} &= \Phi(\cdot) - (A-R)\phi(\cdot)\frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\frac{1}{\phi(\Phi^{-1}(\frac{1+r-v}{1+R-v}))}\frac{1+r-v}{(1+R-v)^2} \\
\frac{\partial I}{\partial r} &= (A-R)\phi(\cdot)\frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\frac{1}{\phi(\Phi^{-1}(\frac{1+r-v}{1+R-v}))} > 0 \\
\frac{\partial I}{\partial \bar{c}} &= (A-R)\phi(\cdot)\frac{\alpha}{\sqrt{\beta}} > 0 \\
\frac{\partial I}{\partial v} &= (A-R)\phi(\cdot)\frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\frac{1}{\phi(\Phi^{-1}(\frac{1+r-v}{1+R-v}))}\frac{r-R}{(1+R-v)^2} < 0,
\end{aligned}$$

where $\Phi(\cdot) = \Phi\left(\frac{\alpha}{\sqrt{\beta}}\left[c^* - \bar{c} - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}\left(\frac{1+r}{1+R}\right)\right]\right)$. By the implicit function theorem we can write:

$$\begin{aligned}
\frac{dc^*}{dR} &= -\frac{\partial I/\partial R}{\partial I/\partial c^*} \\
\frac{dc^*}{dr} &= -\frac{\partial I/\partial r}{\partial I/\partial c^*} < 0 \\
\frac{dc^*}{d\bar{c}} &= -\frac{\partial I/\partial \bar{c}}{\partial I/\partial c^*} < 0 \\
\frac{dc^*}{dv} &= -\frac{\partial I/\partial v}{\partial I/\partial c^*}\frac{\partial c^*}{\partial v} > 0.
\end{aligned}$$

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